# New Approximation Guarantees for the Inventory Staggering Problem

Danny Segev
Tel-Aviv University
Joint work with Noga Alon (Princeton)

SCHEDULING SEMINAR, JUNE 2026

MODEL
NEW RESULTS
HIGH-LEVEL IDEAS

ANALYSIS
METHODS
TECHNICAL DETAILS

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# OUTLINE

$\boxed{\text{MODEL FORMULATION}}$

IMPOSSIBILITY RESULTS

OLD/NEW APPROXIMATIONS

CONCLUDING REMARKS

# SINGLE-ITEM POLICIES

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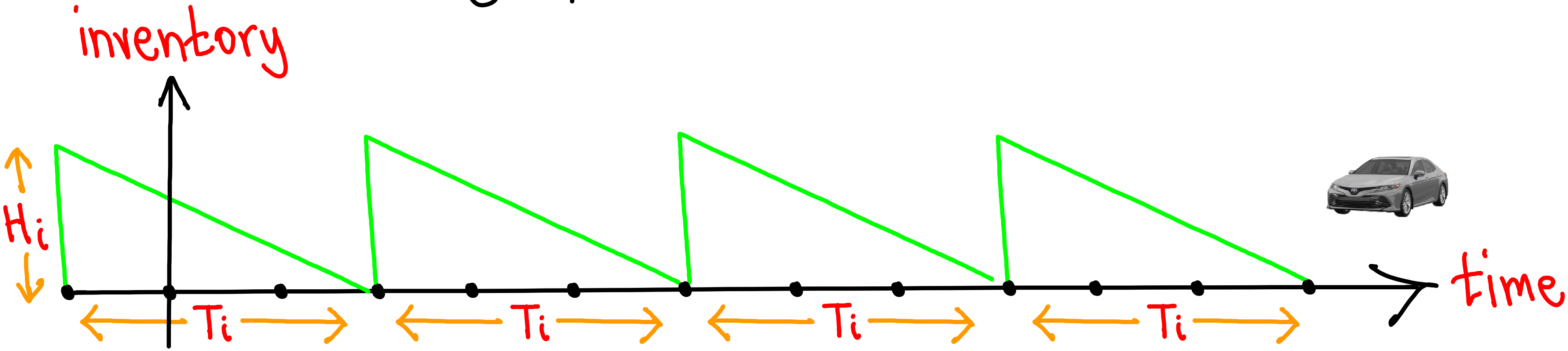
- For each item $i$ ...
  - $T_i$ = time between successive orders (integer-valued)
  - $H_i$ = ordering quantity

INPUT



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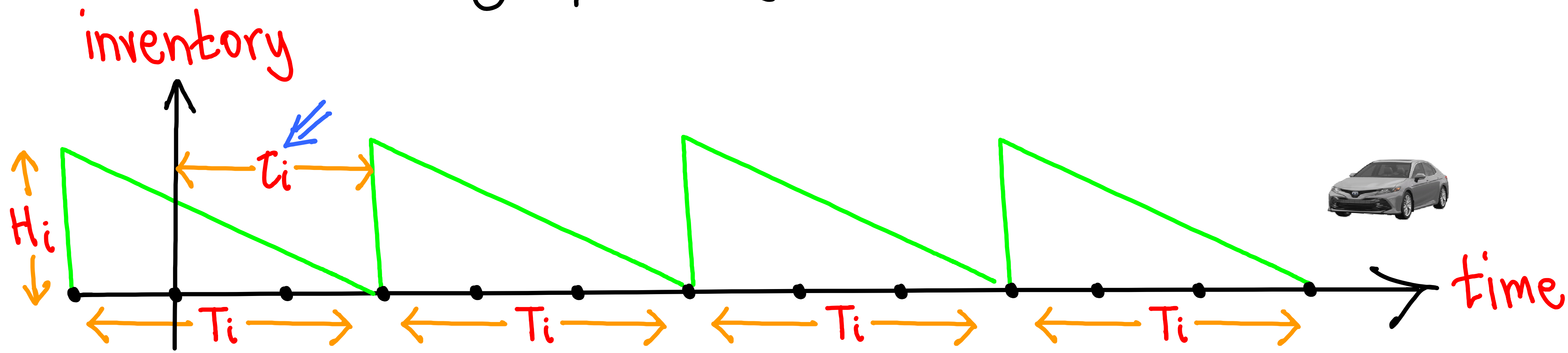


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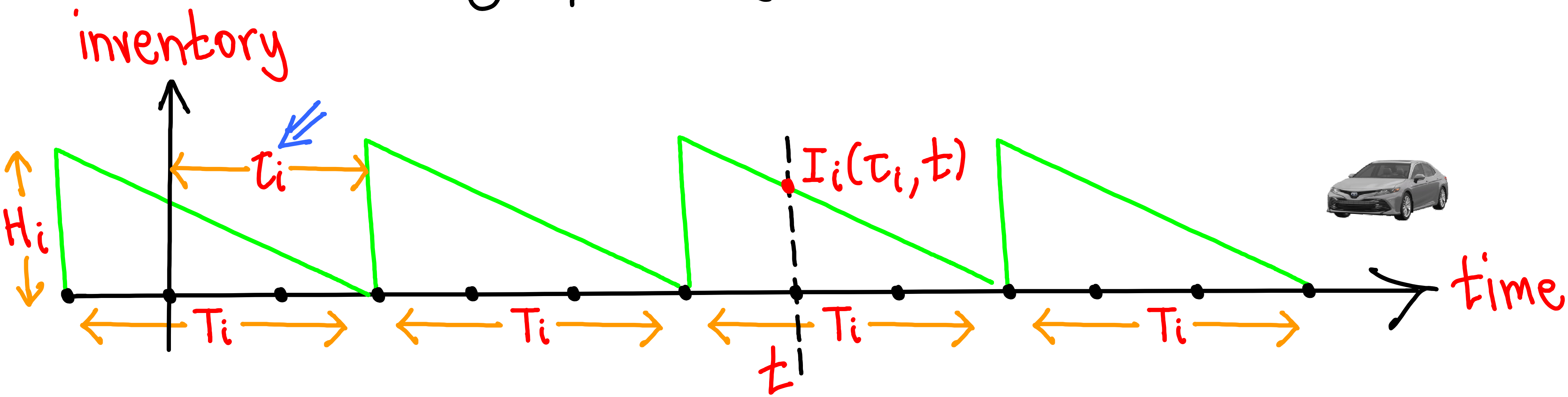


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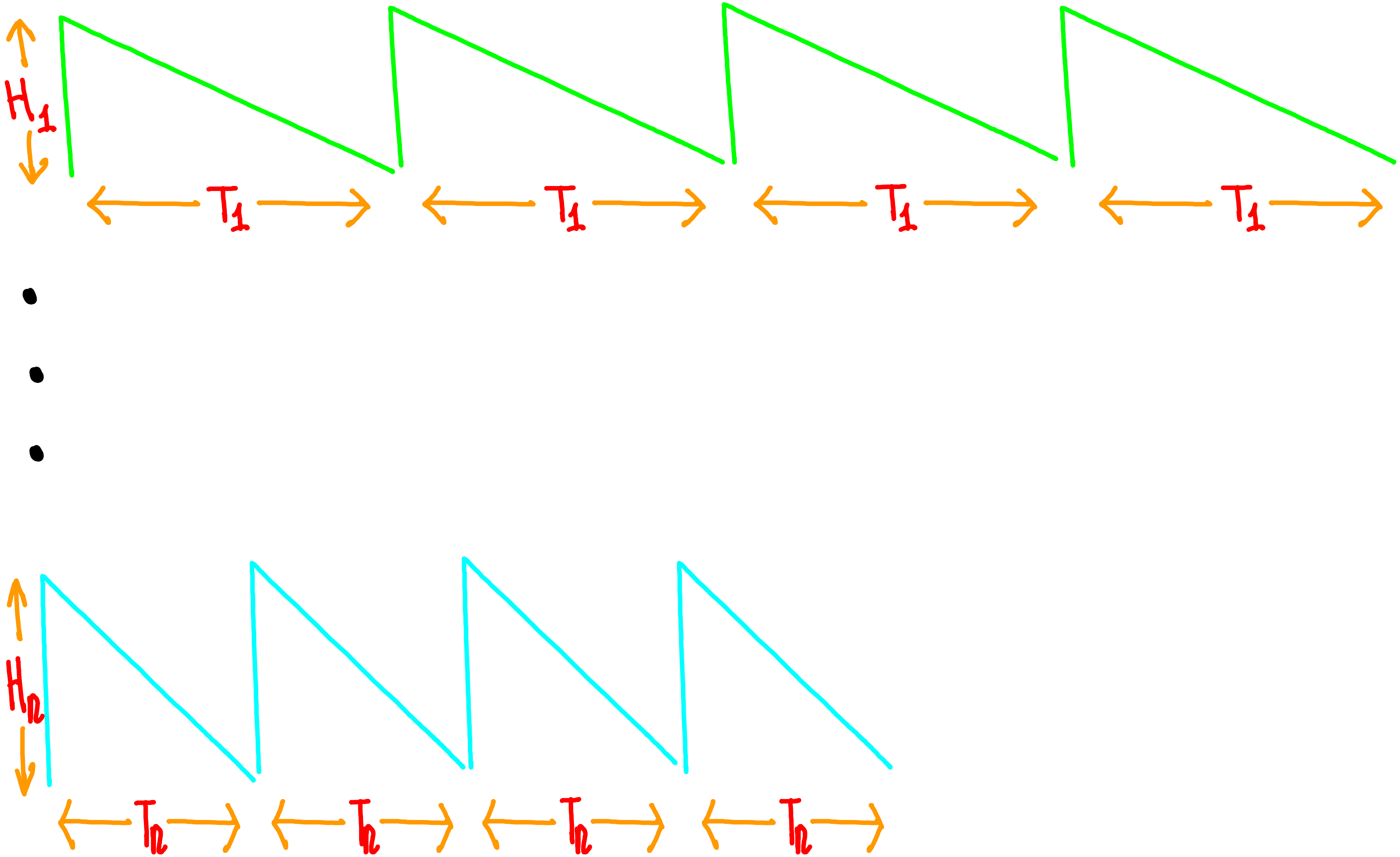

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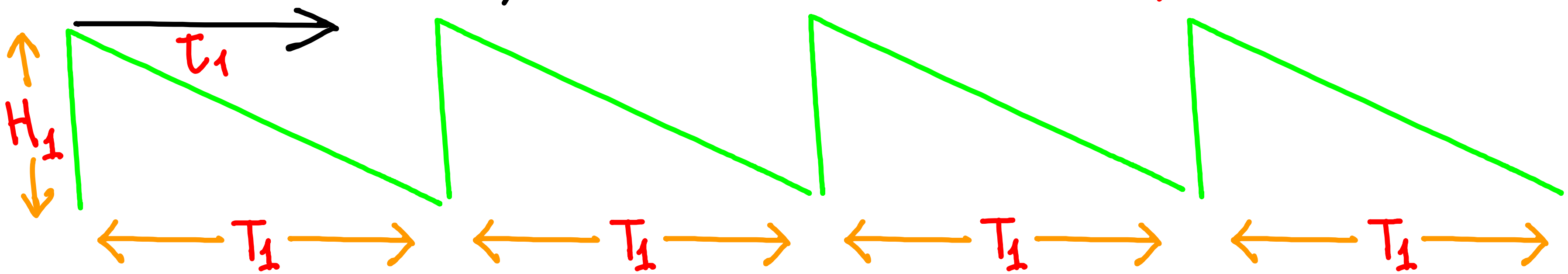


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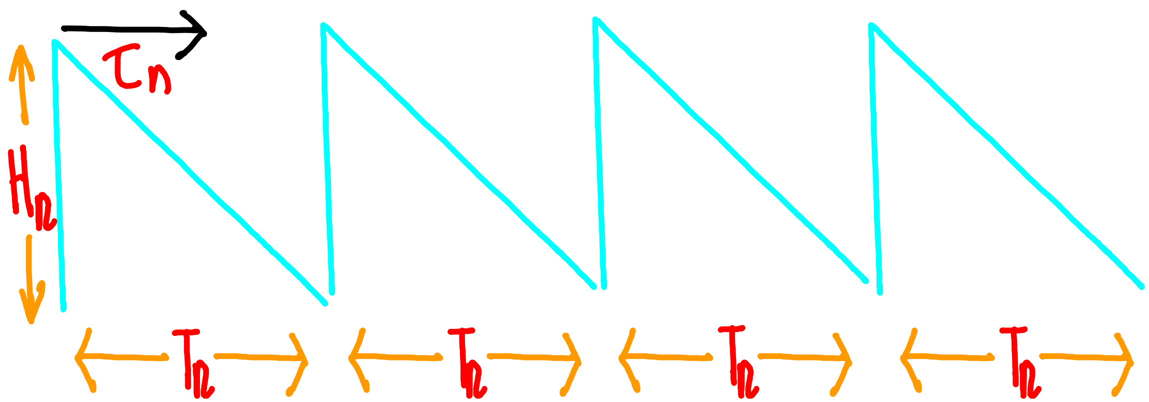

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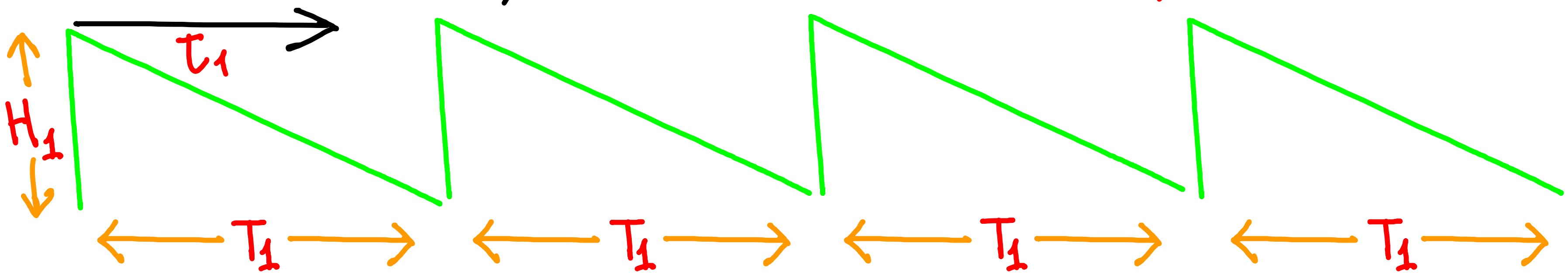


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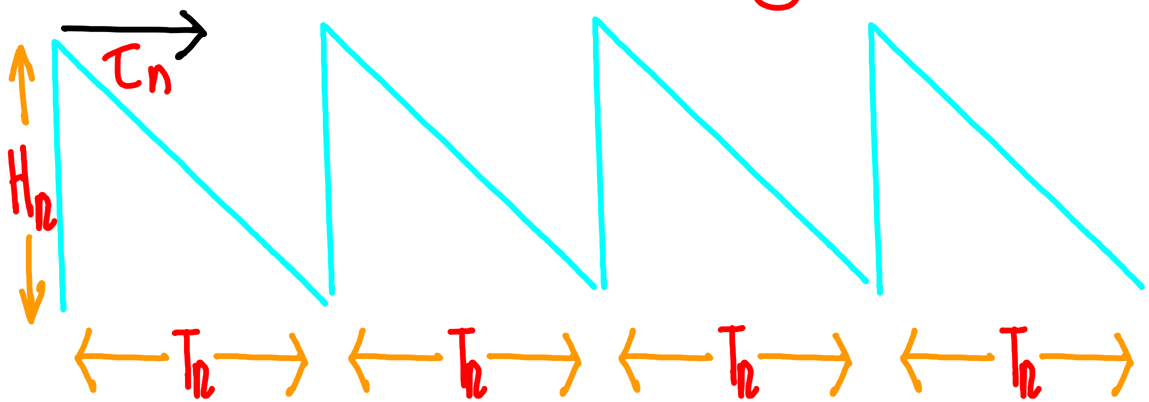

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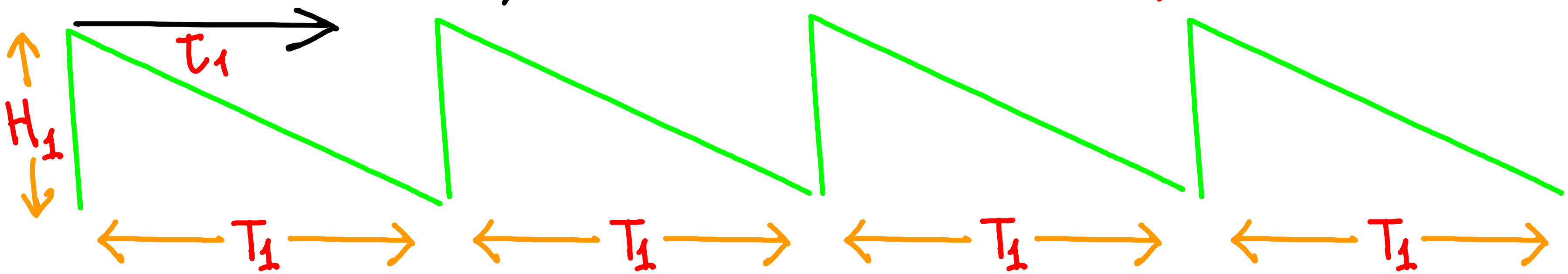


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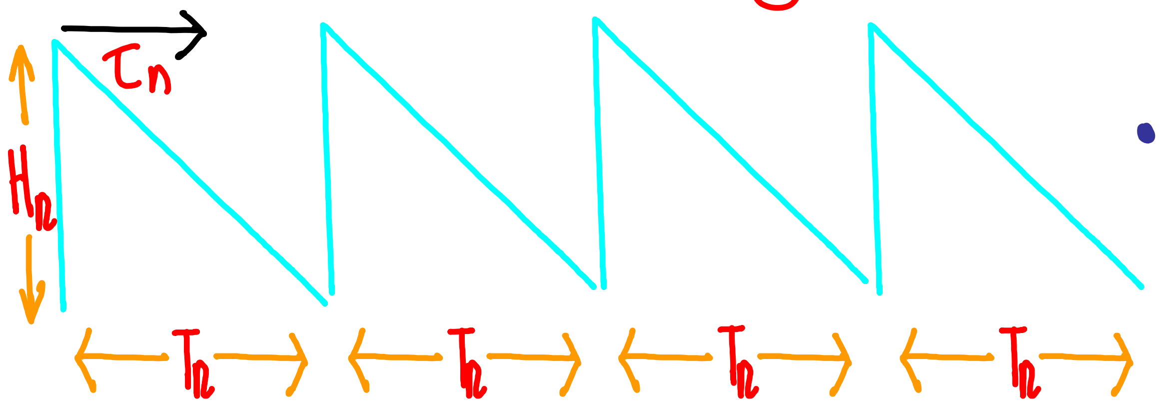


- Peak inventory?

$$I_{max}(\tau) = \max_{t \in \mathbb{Z}} I_\Sigma(\tau, t)$$

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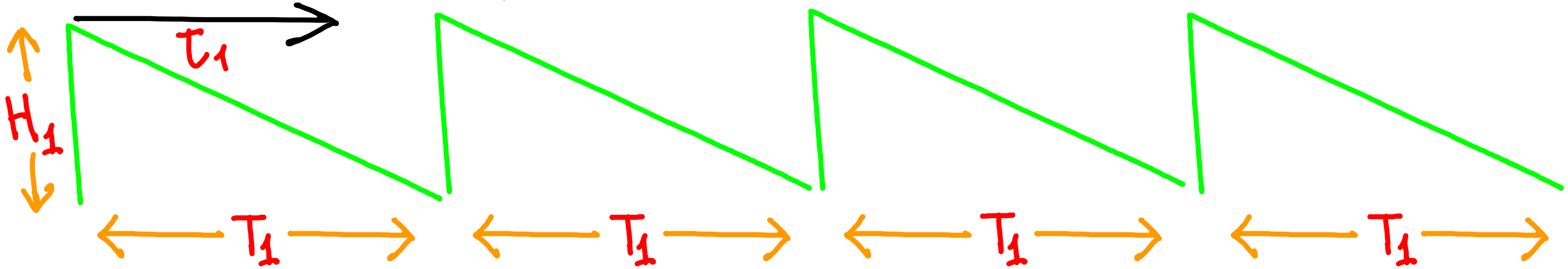


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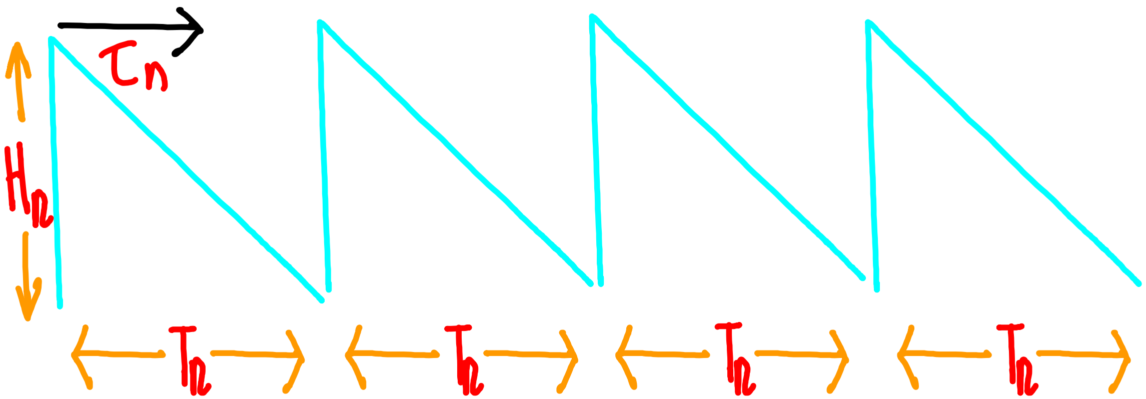
- OBSERVATION: Overall policy has a cycle time of $\Delta = \mathrm{LCM}(T_1, \dots, T_n)$, so $I_{max}(\tau) = \max_{t \in [0,\Delta]} I_\Sigma(\tau, t)$



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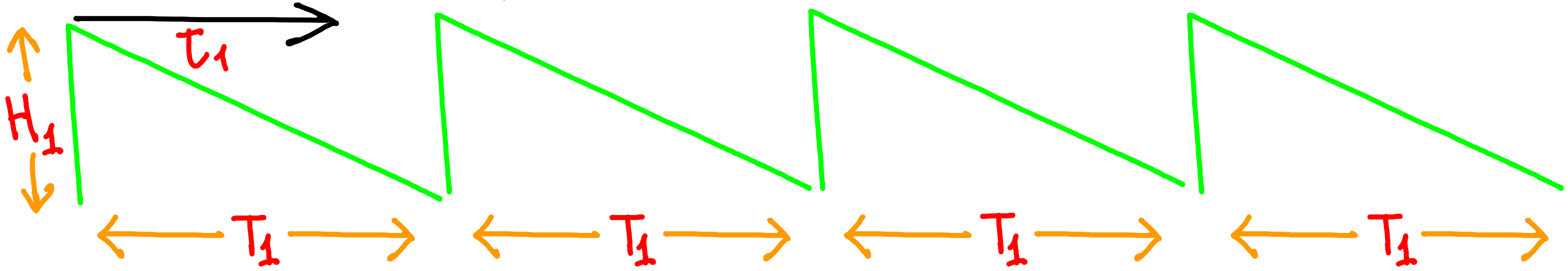


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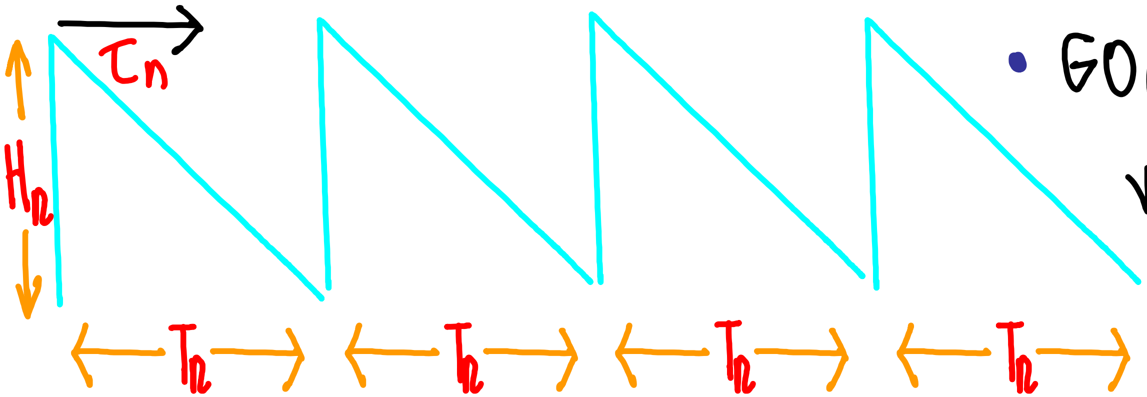
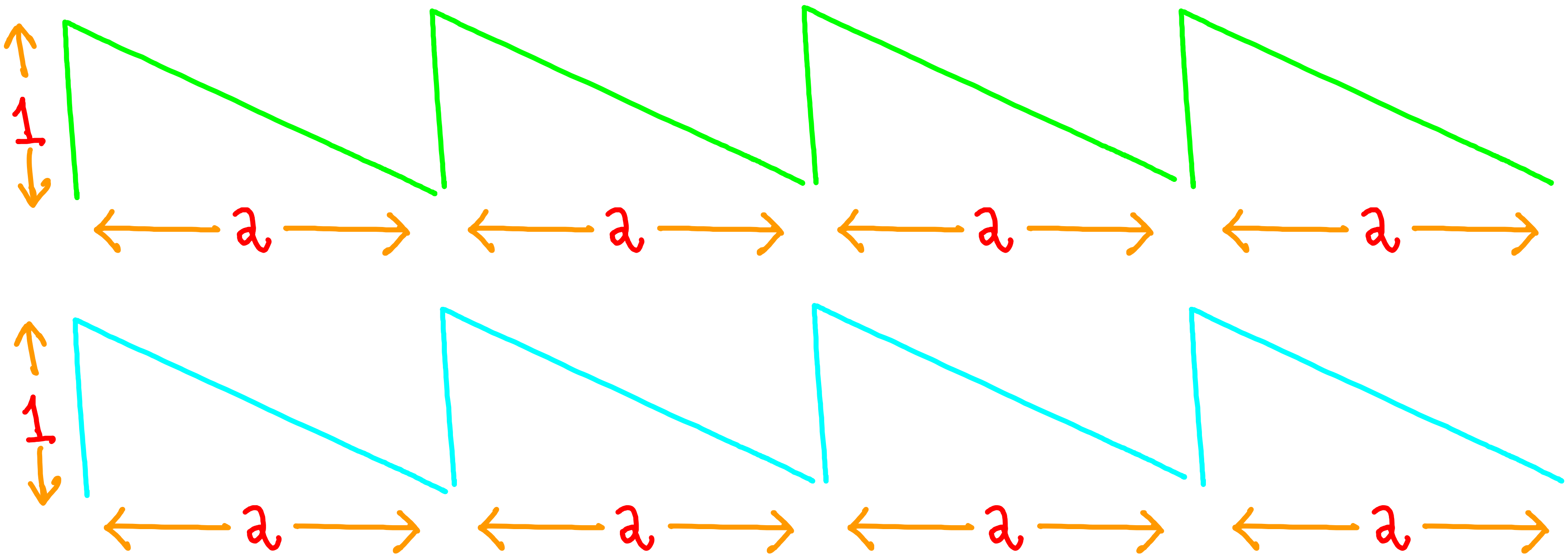



- GOAL: Compute shift vector $\tau$ with minimum peak inventory $I_{max}(\tau)$

## SIMPLE EXAMPLE

- Two identical items...



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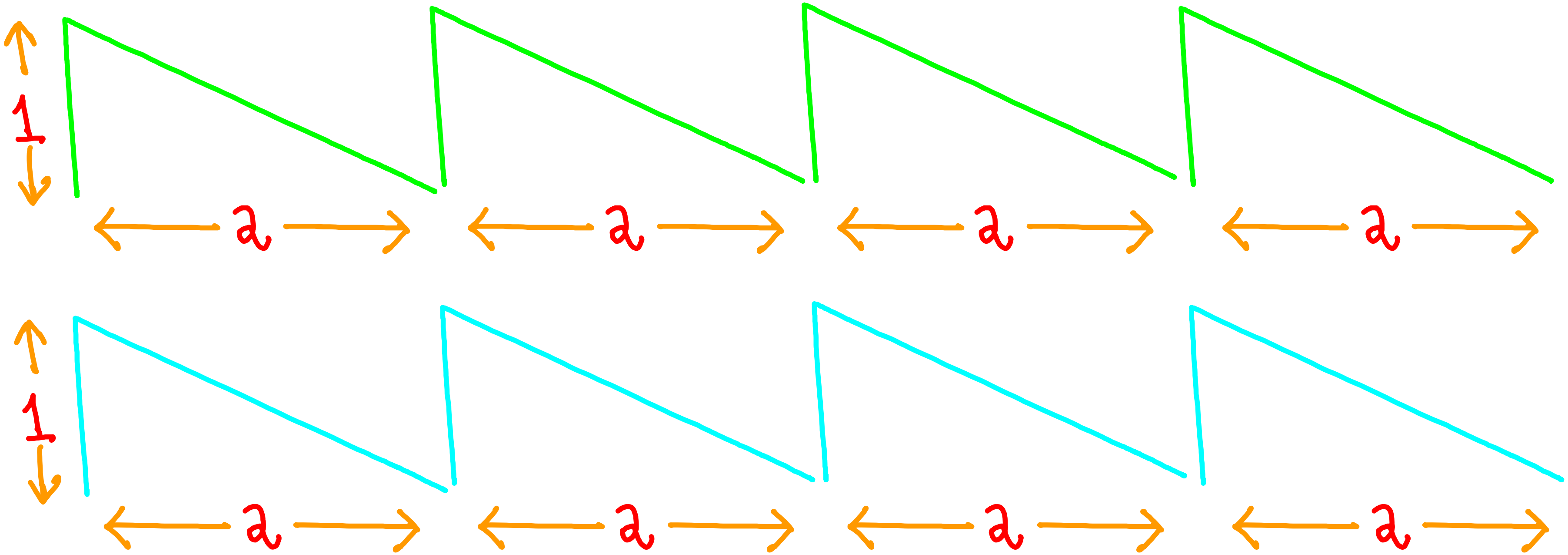


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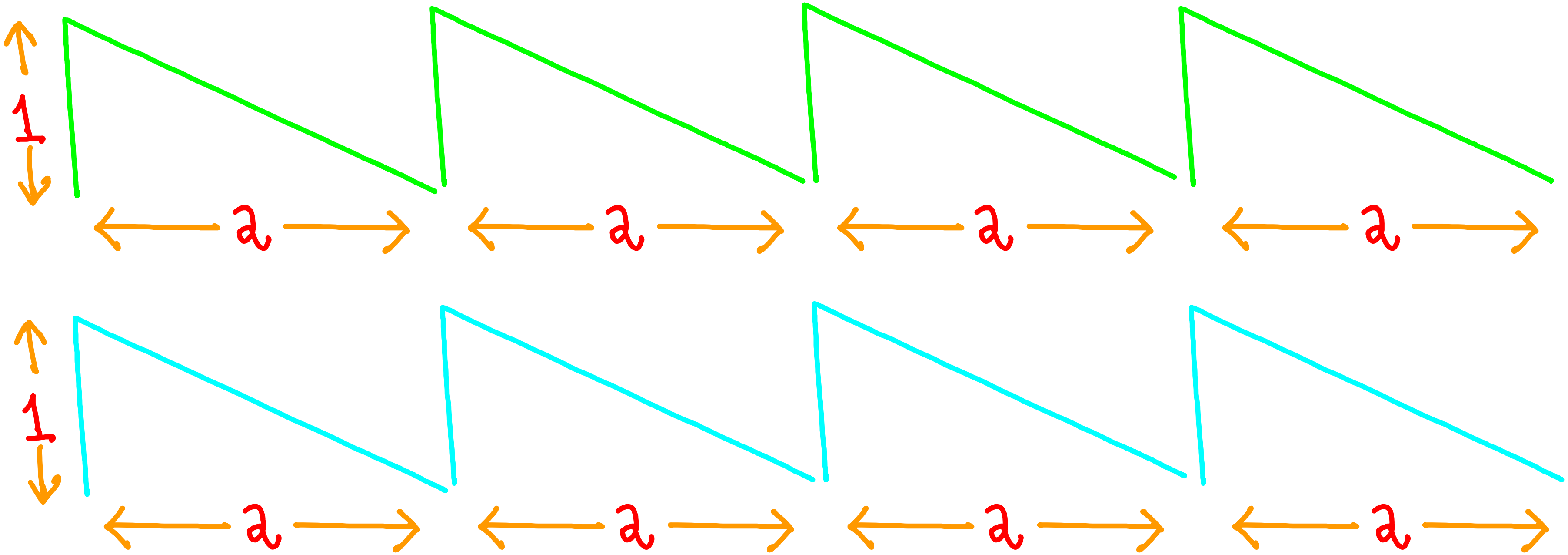


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$$t_1 = 0\,, \quad t_2 = 1 \quad \Rightarrow \quad I_{max}(0,1) = 3/a$$

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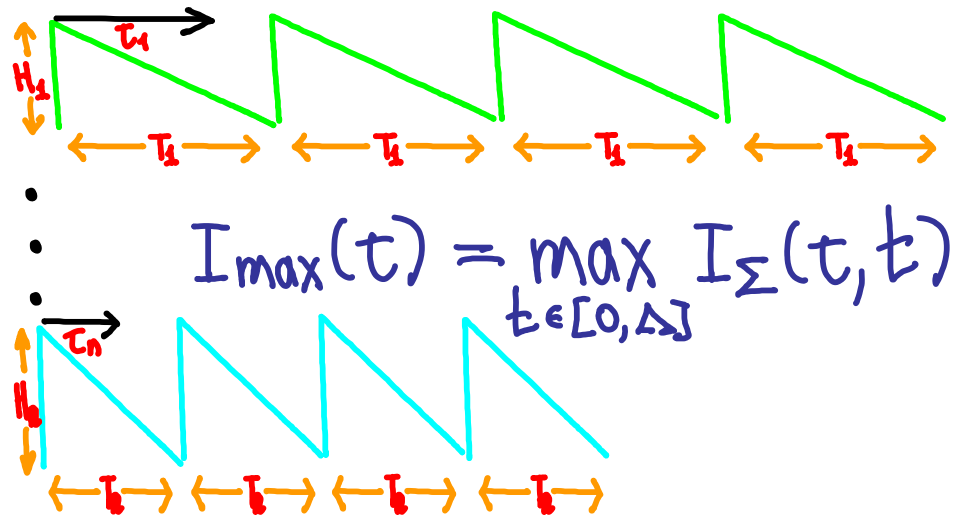
# (UN)KNOWN HARDNESS RESULTS

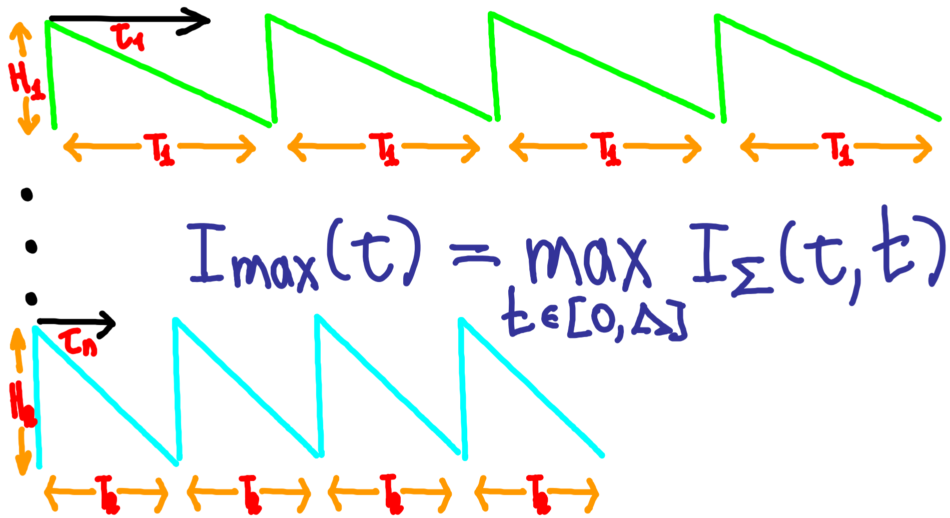



$$I_{max}(\tau) = \max_{t \in [0,\Delta]} I_{\Sigma}(\tau, t)$$

# (UN)KNOWN HARDNESS RESULTS

- Continuous-time formulation is strongly NP-hard

  [Gallego, Shaw, Simchi-Levi '92]



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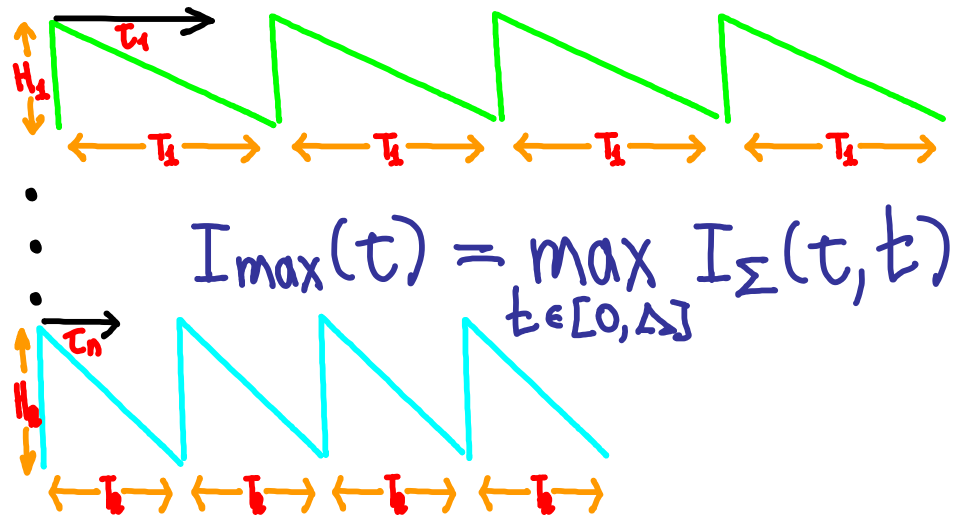
- Discrete setting is ...
  - weakly NP-hard even when $T_1 = \cdots = T_n = 2$ [Hall '98]
  - strongly NP-hard in general [Hochbaum, Rao '19]

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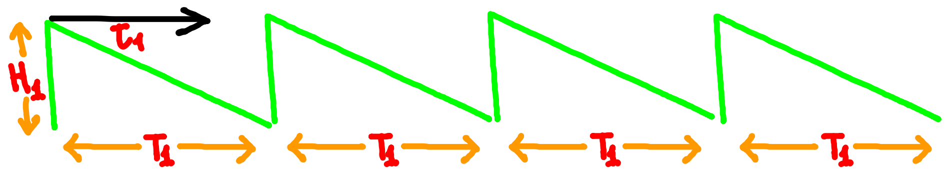
- BASIC OPEN QUESTION:
  Given a shift vector $\tau$, evaluate its peak inventory $I_{max}(\tau)$ in polynomial time?

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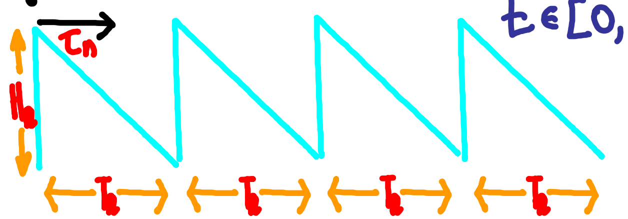
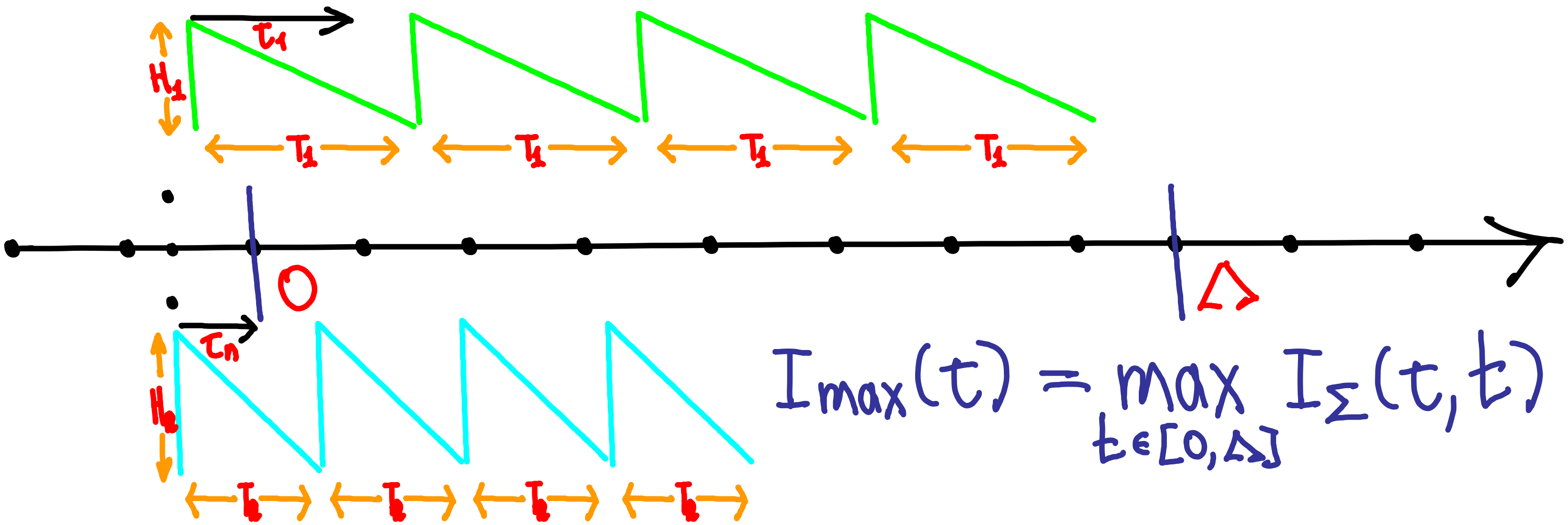


# PEAK EVALUATION BY SAMPLING?

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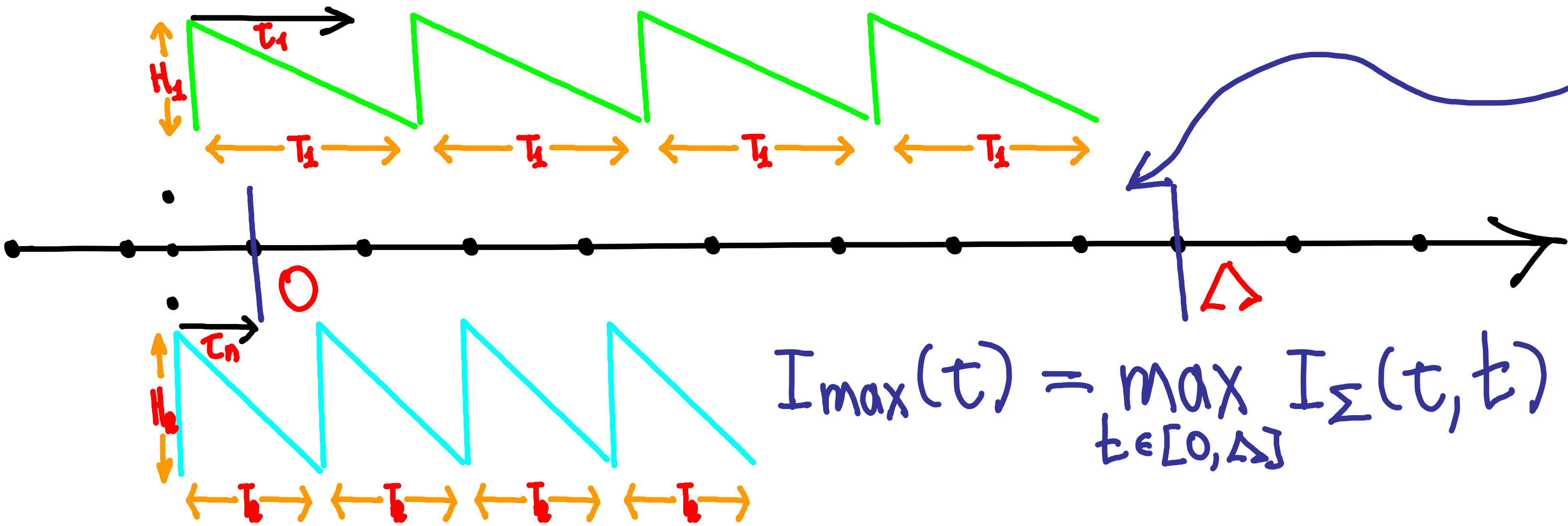
- Independently draw M points $X_1, \dots, X_M \sim U\{1, \dots, \Delta\}$

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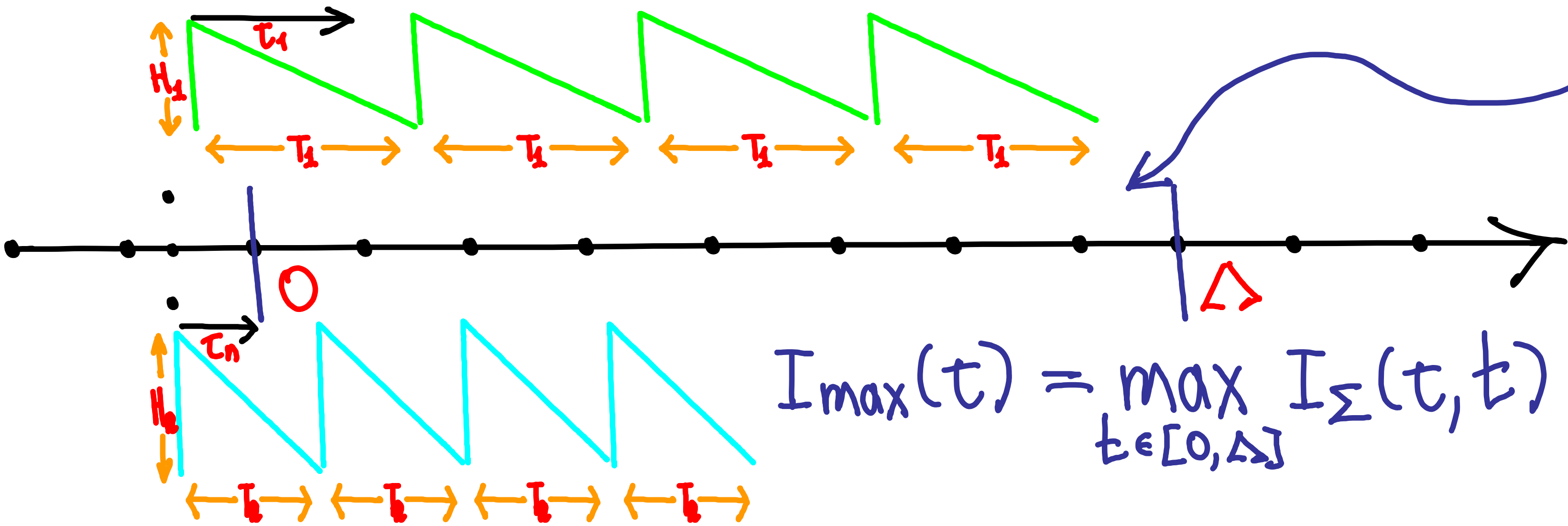


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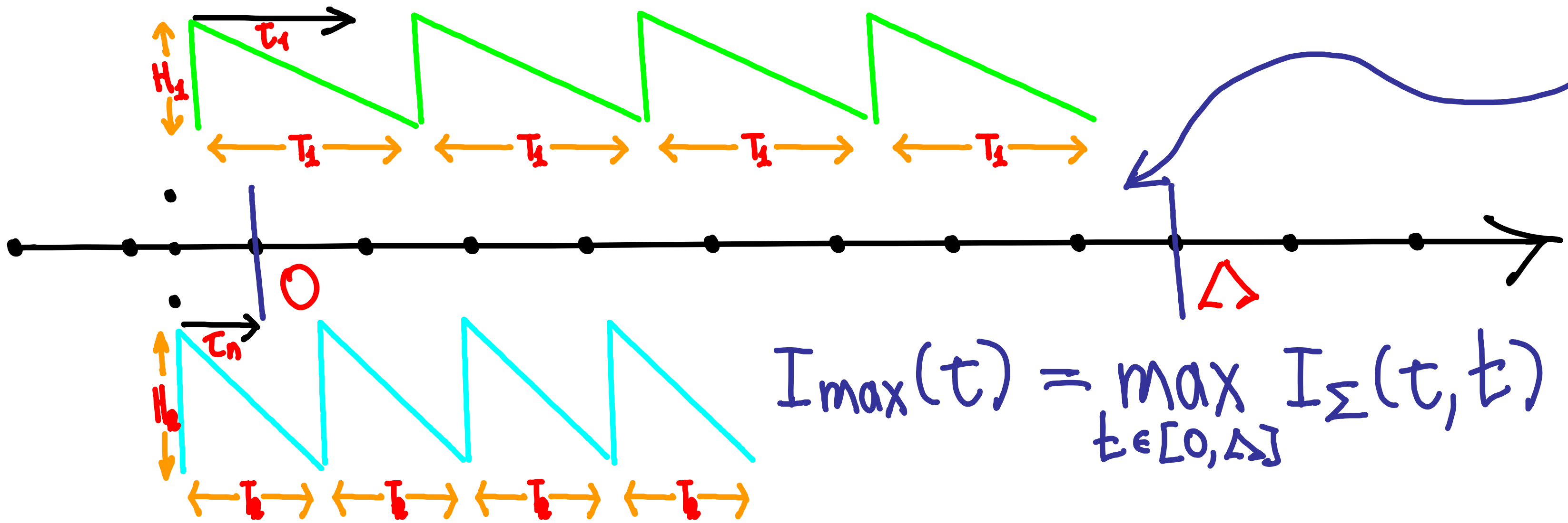


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- QUESTION: Required number of points $M$ to ensure

$$\tilde{I}^M_{max} \approx I_{max}(t)$$

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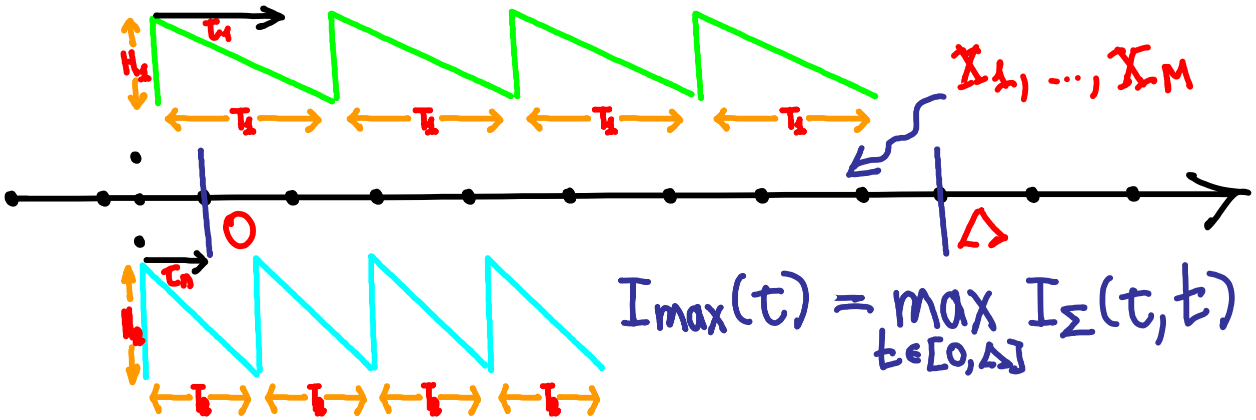
- THEOREM: Exist instance and shift vector $\tau$ for which

$$\Pr\left[\tilde{I}^{M}_{max} \geq \left(\tfrac{1}{2}+\varepsilon\right)\cdot I_{max}(\tau)\right] \leq M\cdot e^{-\varepsilon^2 n/6}$$

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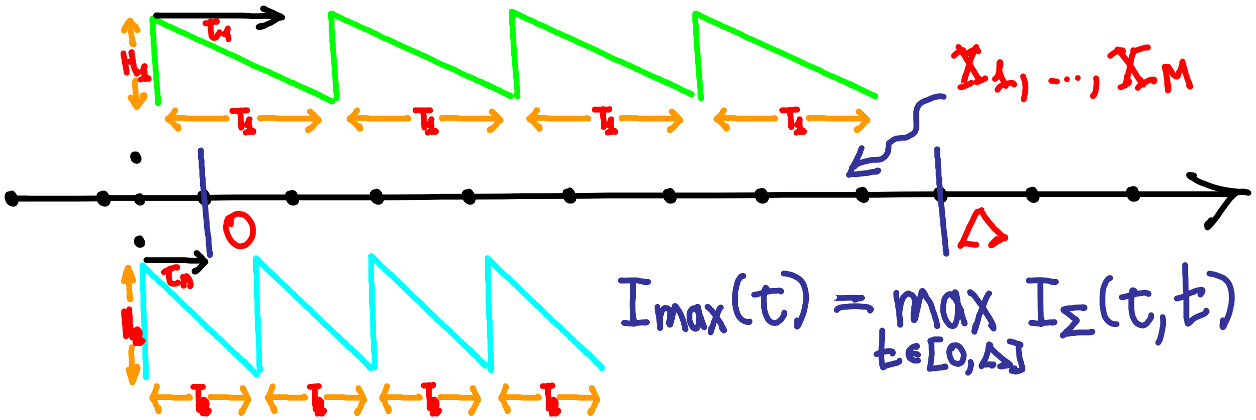

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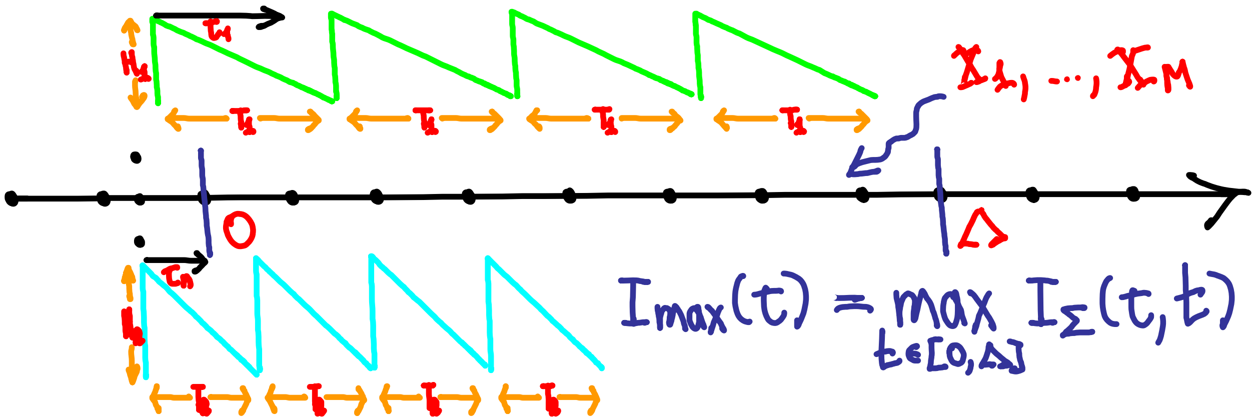


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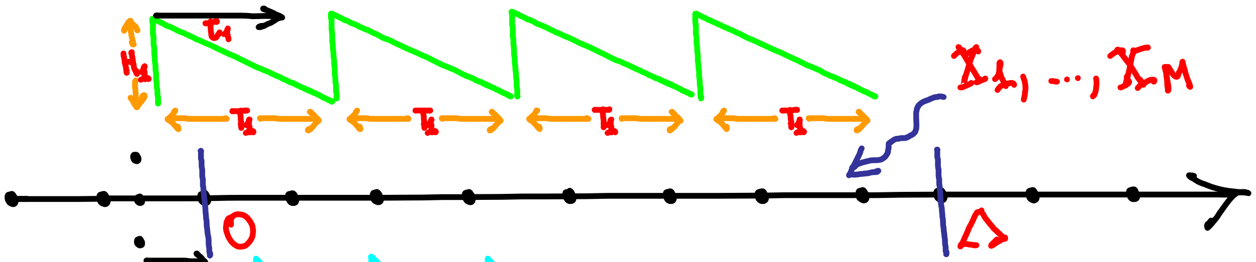


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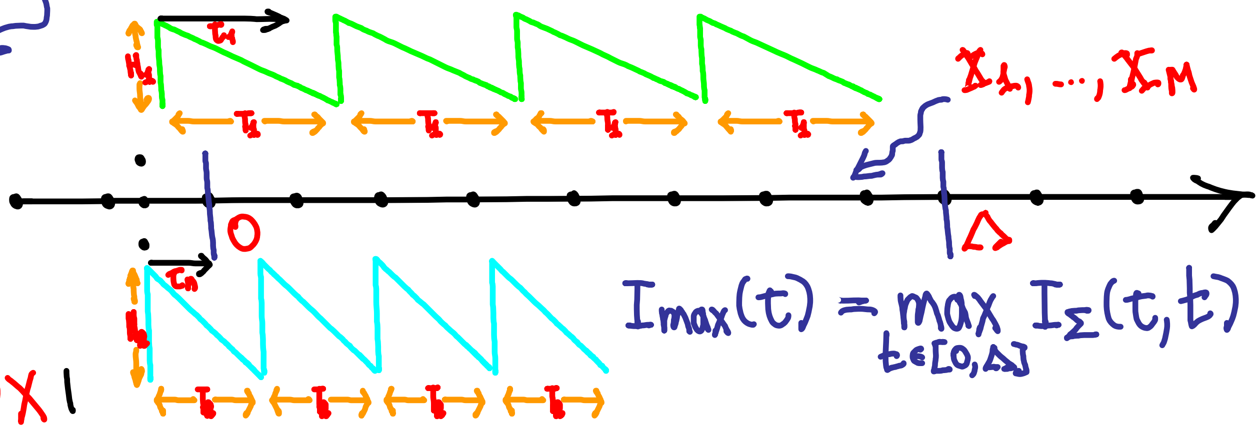
average-space bound:

$$OPT \geq \frac{1}{2}\cdot\sum_{i\in[n]} H_i$$

any vector is 2-approx!

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want $I_\Sigma(\tau, X_m)$ concentrated around $\mathbb{E}[I_\Sigma(\tau, X_m)]$

but $I_\Sigma(\tau, X_m) = \sum_{i\in[n]} I_i(\tau_i, X_m)$ generally correlated

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$I_\Sigma(\tau, X_m)$ concentrated around $\approx n/2$ while $I_{max}(\tau) = n$

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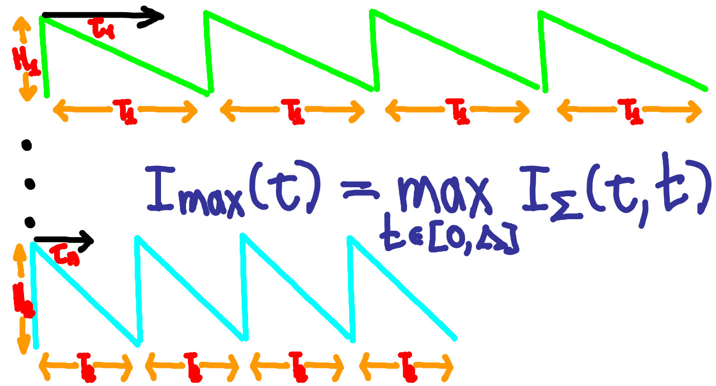

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# GLOBAL NATURE: GROUPWISE SYNCHRONIZATION?

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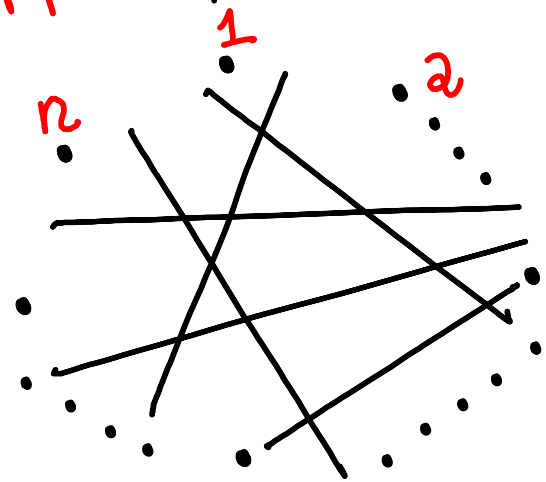
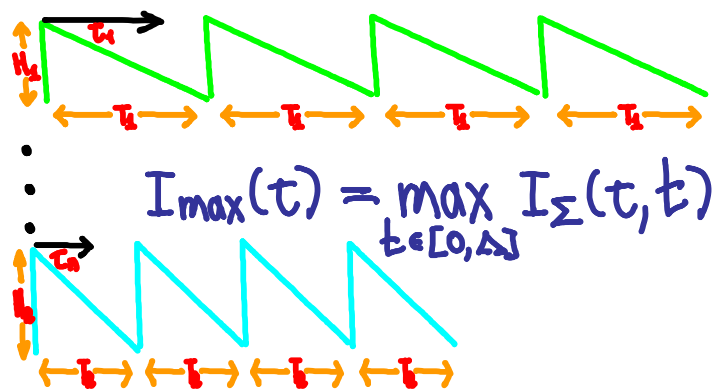
- CONJECTURE: When $OPT \leq (1-\delta) \cdot \sum_{i \in [n]} H_i$ pairwise synchronization should guarantee $I_{max}(\tau) \leq (2 - f(\delta)) \cdot OPT$



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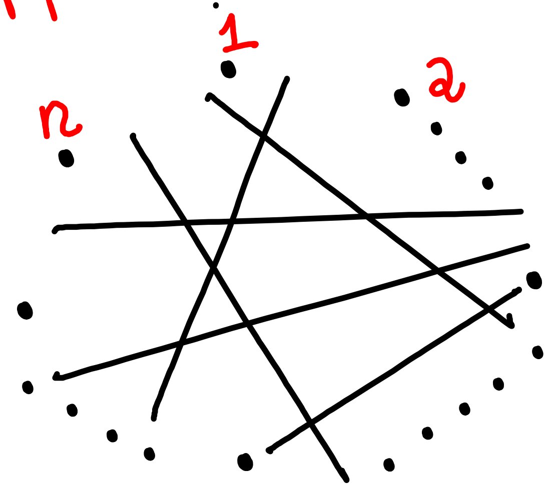


- CONJECTURE: When $OPT \leq (1-\delta) \cdot \sum_{i \in [n]} H_i$ pairwise synchronization should guarantee $I_{max}(\tau) \leq (2 - f(\delta)) \cdot OPT$

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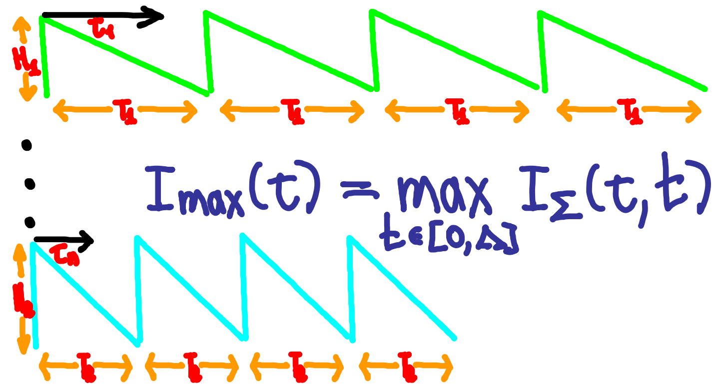

# GLOBAL NATURE: GROUPWISE SYNCHRONIZATION?

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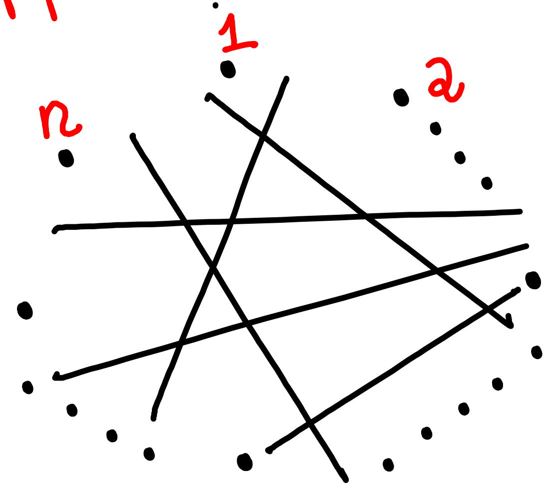


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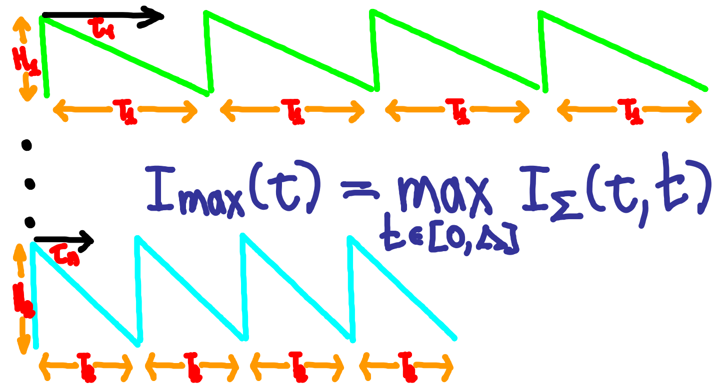

$$I_{max}(\tau) = \max_{t \in [0,\Delta]} I_{\Sigma}(\tau, t)$$

- Beyond pairs: Subsets of size $\leq 3$? $\leq 4$? … ?

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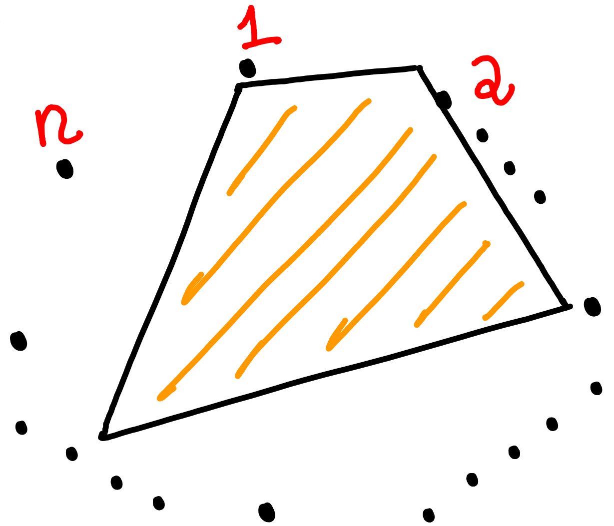


- Beyond pairs: $\leq 3$? $\leq 4$? $\cdots$?

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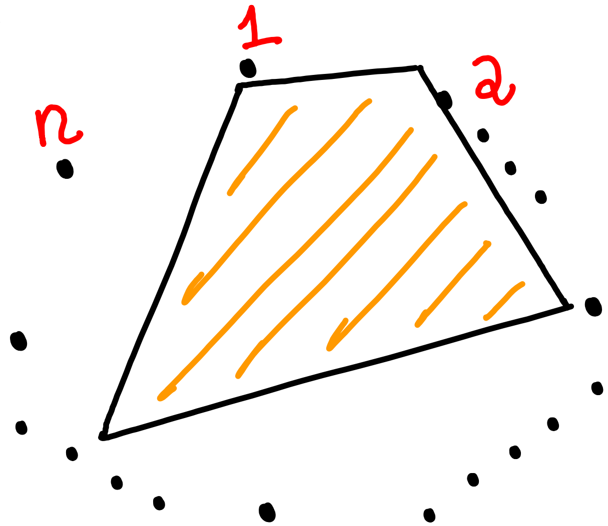


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  - Construction based on combinatorial designs
  - Proved interesting generalization of CRT

# OUTLINE

MODEL FORMULATION

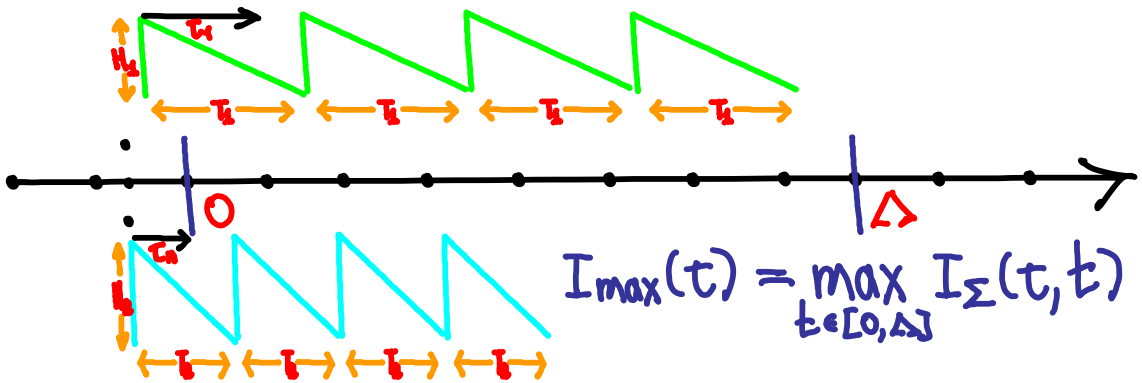
IMPOSSIBILITY RESULTS

$\boxed{\text{OLD/NEW APPROXIMATIONS}}$

CONCLUDING REMARKS

cycle length

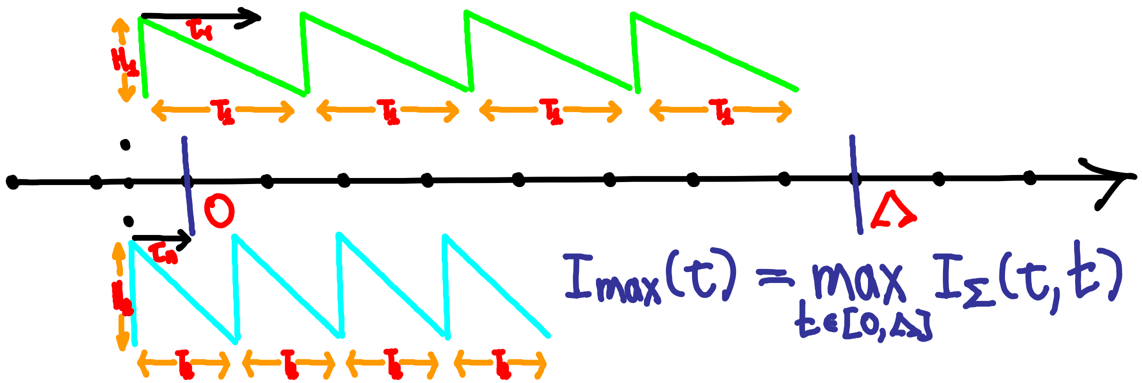
# APPROXIMATION IN TERMS OF Δ




$$I_{max}(\tau) = \max_{t \in [0,\Delta]} I_{\Sigma}(\tau, t)$$

# APPROXIMATION IN TERMS OF $\Delta$ cycle length

- Why sub-2-approx may be possible?
  - At most $\Delta$ options for each shift
  - Don't have peak evaluation issues

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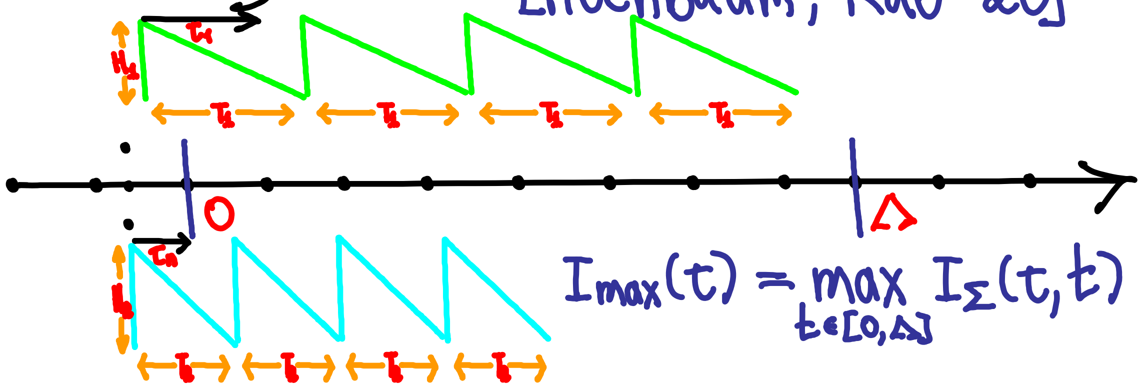


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# APPROXIMATION IN TERMS OF $\Delta$ (cycle length)

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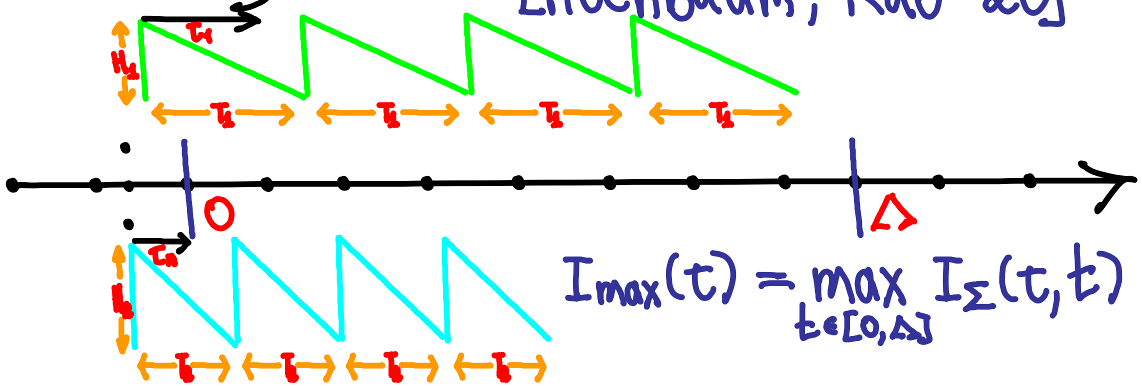
- OPEN QUESTION:
  non-constant $\Delta$?



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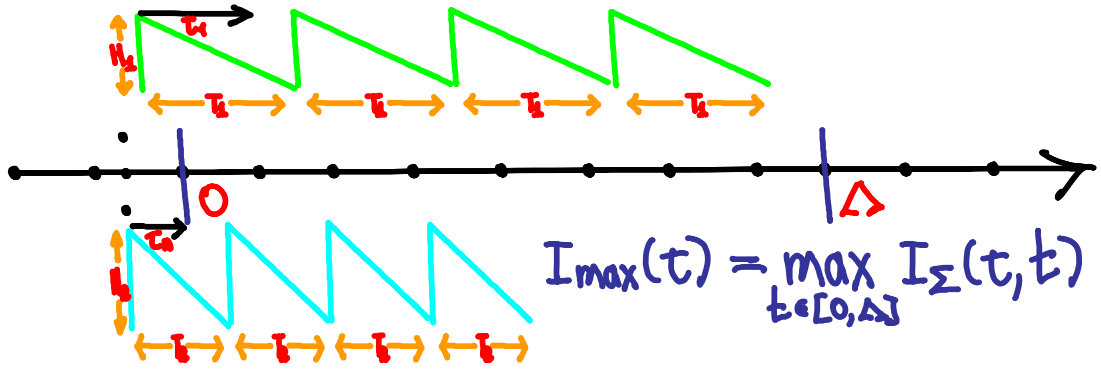
# APPROXIMATION IN TERMS OF $\Delta$

cycle length

- OPEN QUESTION: non-constant $\Delta$?

- THEOREM: Randomized LP-based $(1+\varepsilon)$-approx in time $O(|I|^{O(1)} \cdot \Delta^{O(1/\varepsilon^3)})$

  polynomial in $\Delta$

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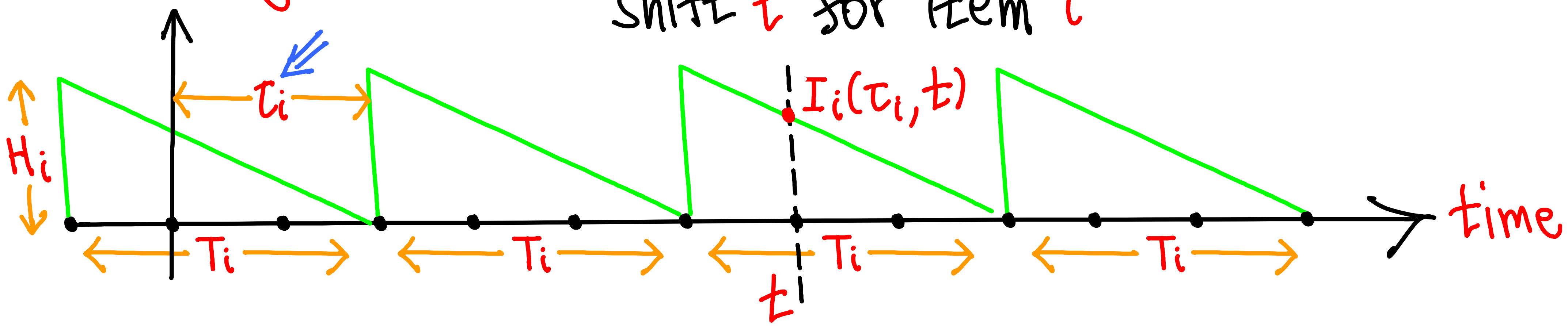
APPROXIMATION IN TERMS OF $\Delta$ cycle length

$X_{i,\tau}$ indicates choosing shift $\tau$ for item $i$



cycle length

# APPROXIMATION IN TERMS OF $\Delta$

$$(LP)\quad \min\ P$$

$$\text{s.t.}\quad \sum_{\tau\in[\Delta]} x_{i,\tau} = 1 \qquad \forall i\in[n]$$

$$\sum_{i\in[n]}\sum_{\tau\in[\Delta]} I_i(\tau,t)\cdot x_{i,\tau} \leq P \qquad \forall t\in[\Delta]$$

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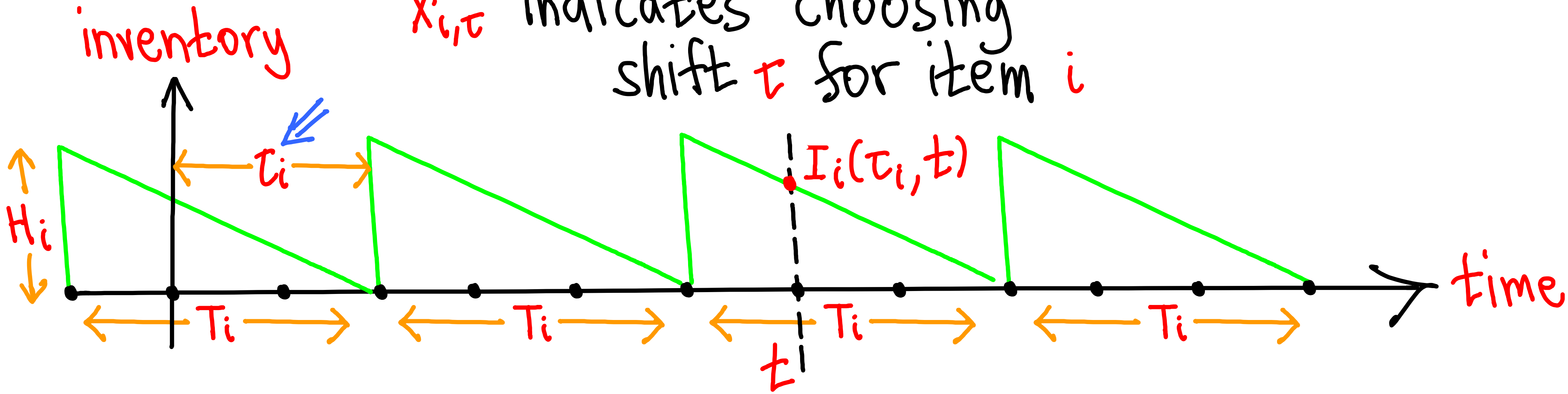

# APPROXIMATION IN TERMS OF $\Delta$

cycle length

integrality gap of $\approx 2$

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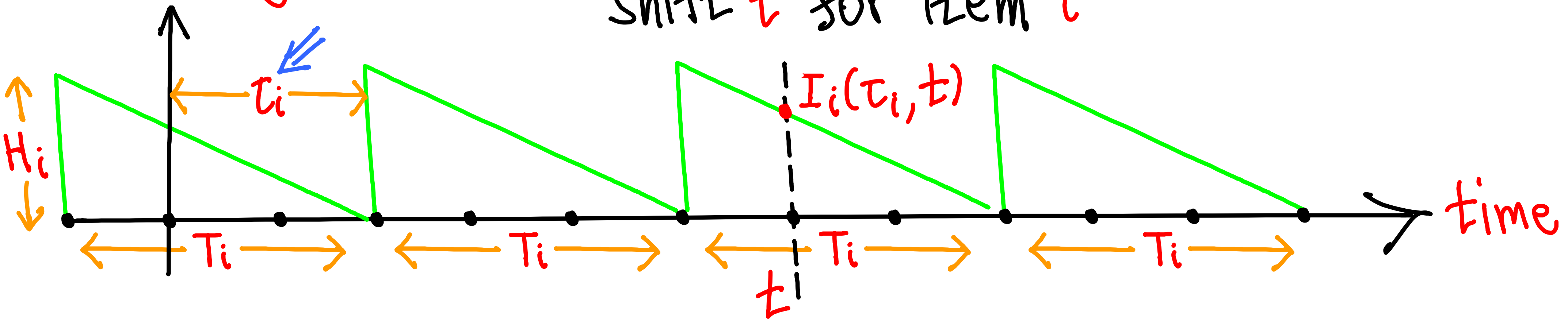

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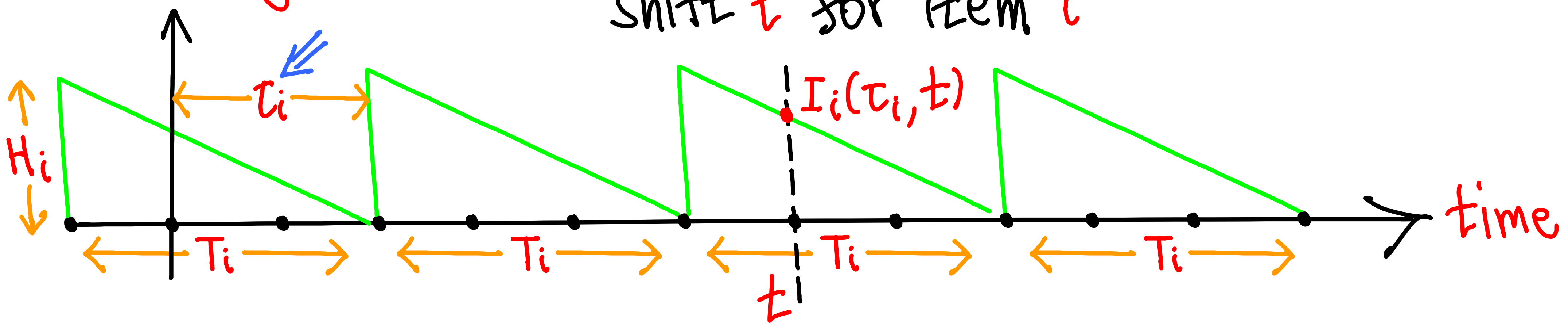
inventory



# APPROXIMATION IN TERMS OF $\Delta$

cycle length

integrality gap of $\approx \cancel{2}$ $1+\varepsilon$

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fix x-values for $\approx \frac{\log \Delta}{\varepsilon^3}$ items

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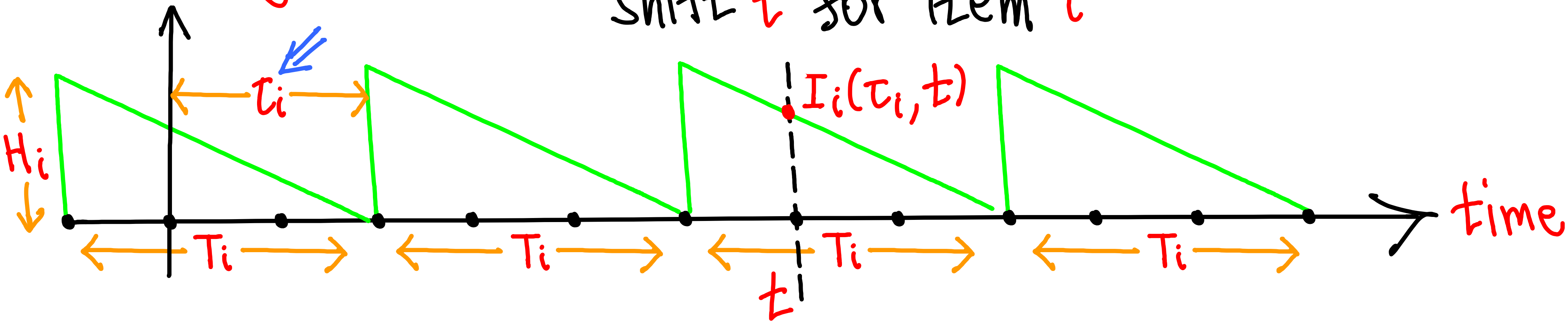

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(LP) $\min P$

Randomized rounding

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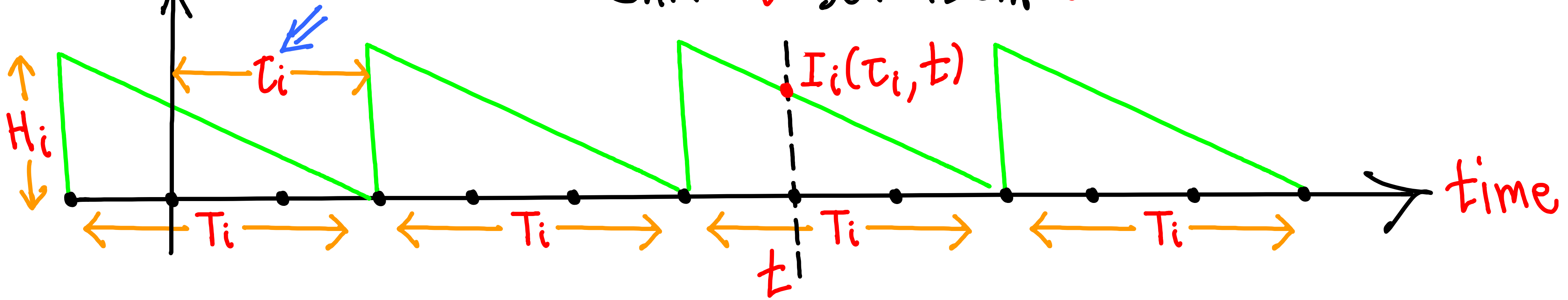
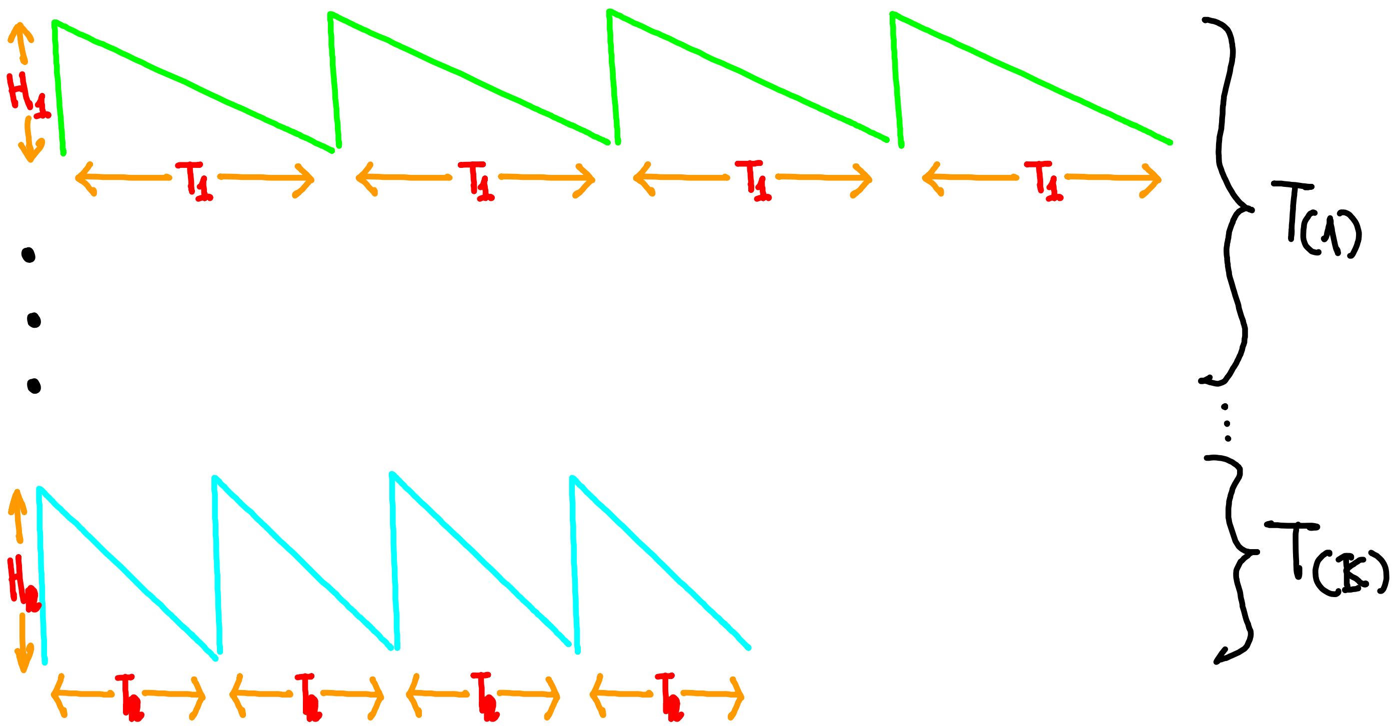


number of distinct times

# APPROXIMATION IN TERMS OF $\underline{K}$



number of distinct times

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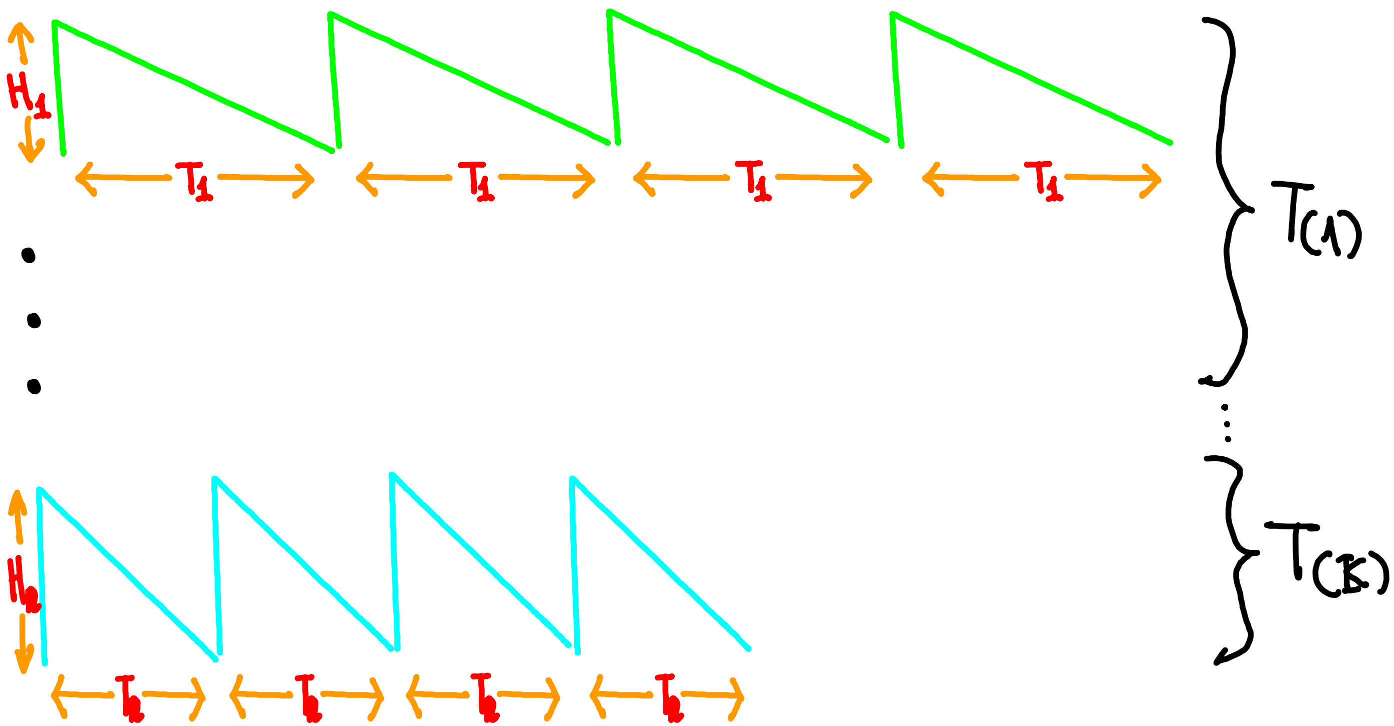

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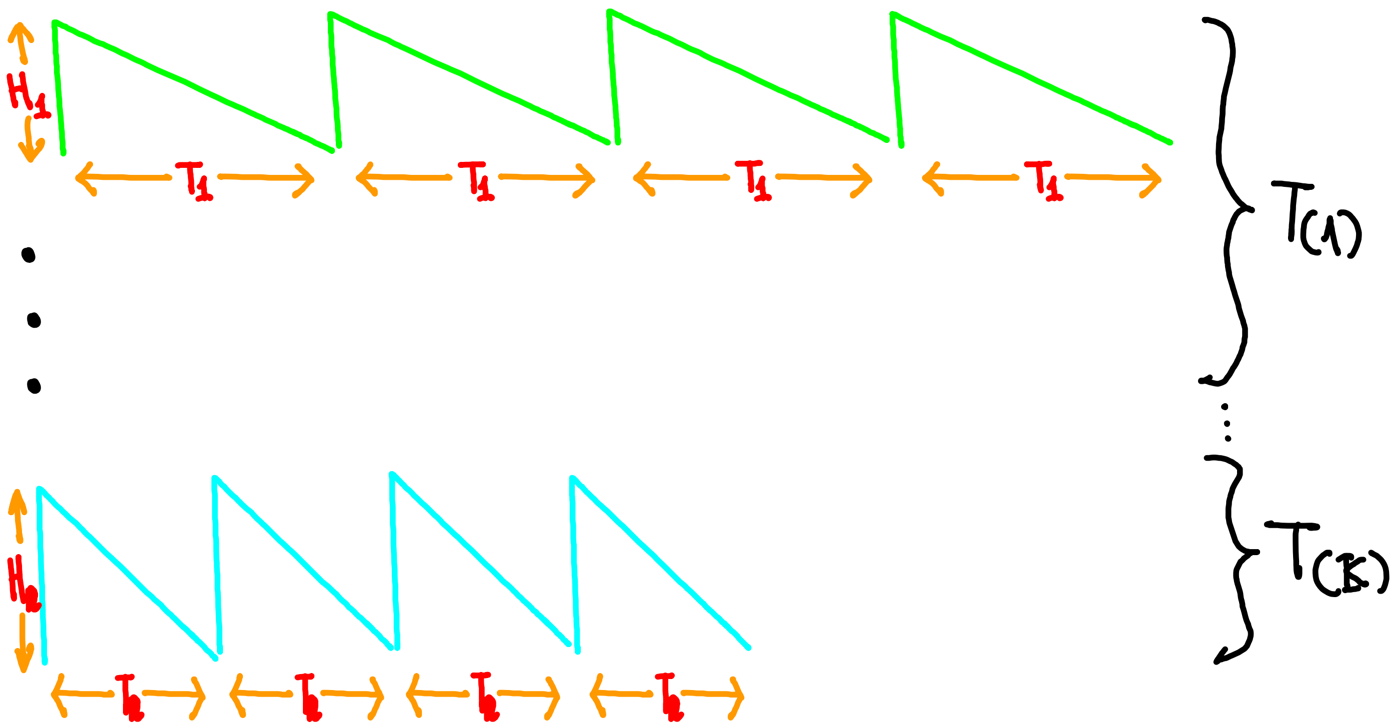

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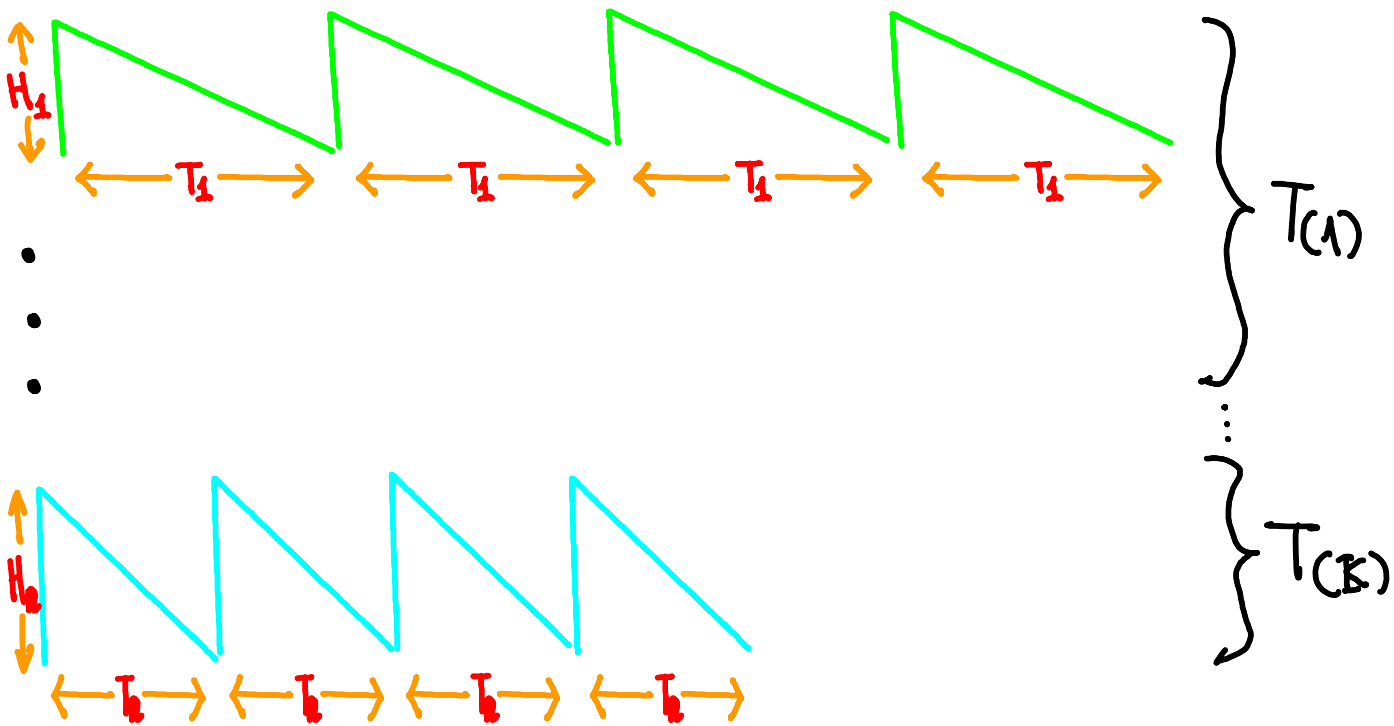
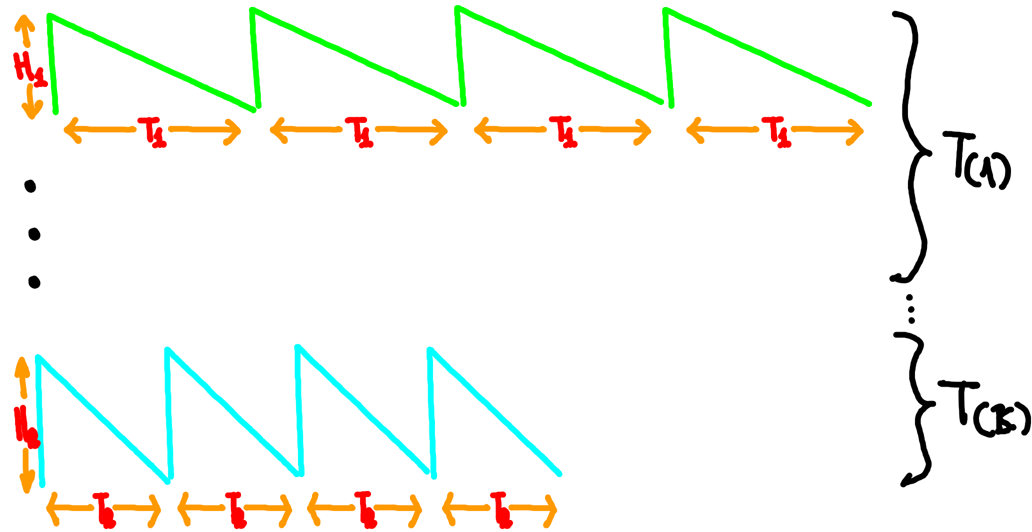



- OPEN QUESTION: Non-trivial result for fixed $K \geq 3$?

number of distinct times

# APPROXIMATION IN TERMS OF $K$

- THEOREM: $(1+\varepsilon)$-approx in $O(|I|^{O(1)} \cdot 2^{\tilde{O}(K/\varepsilon^2)})$ time



number of distinct times

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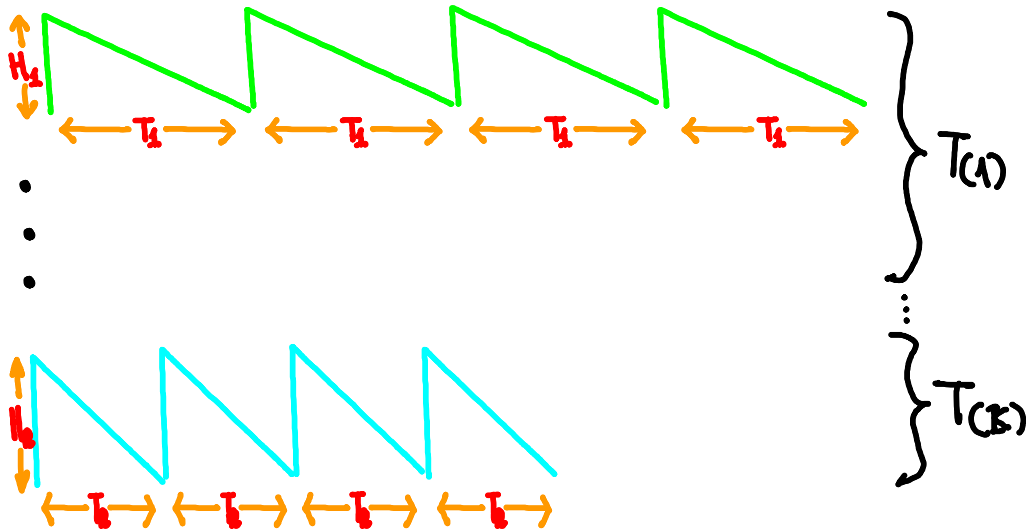

number of distinct times

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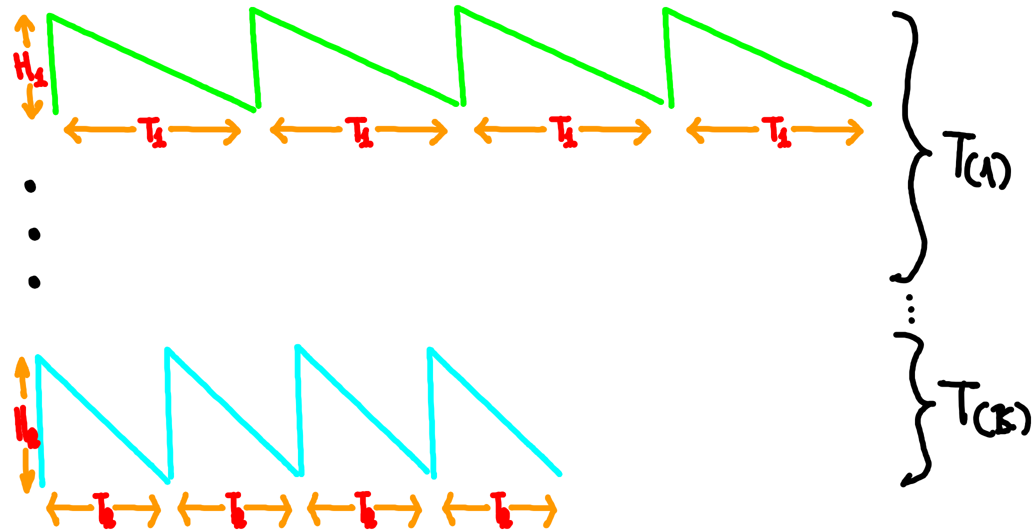

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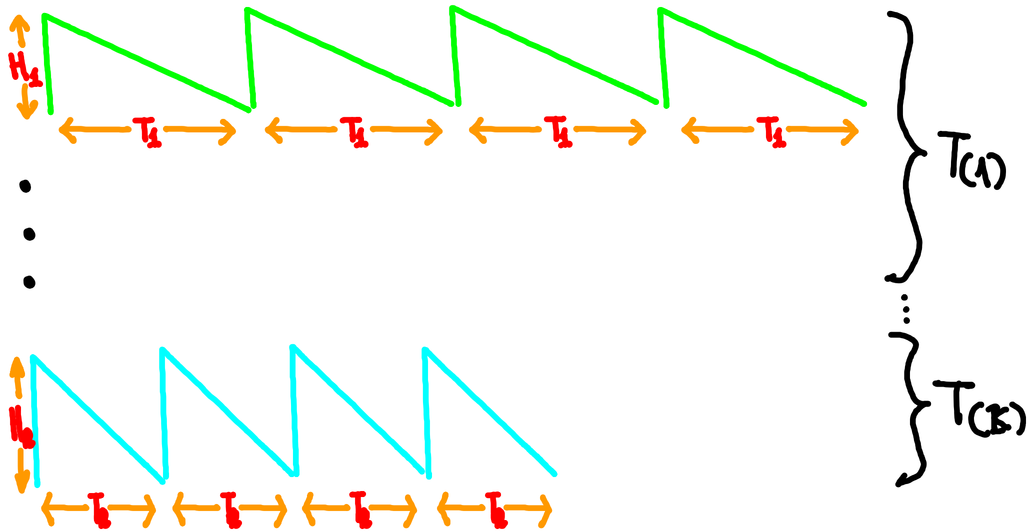
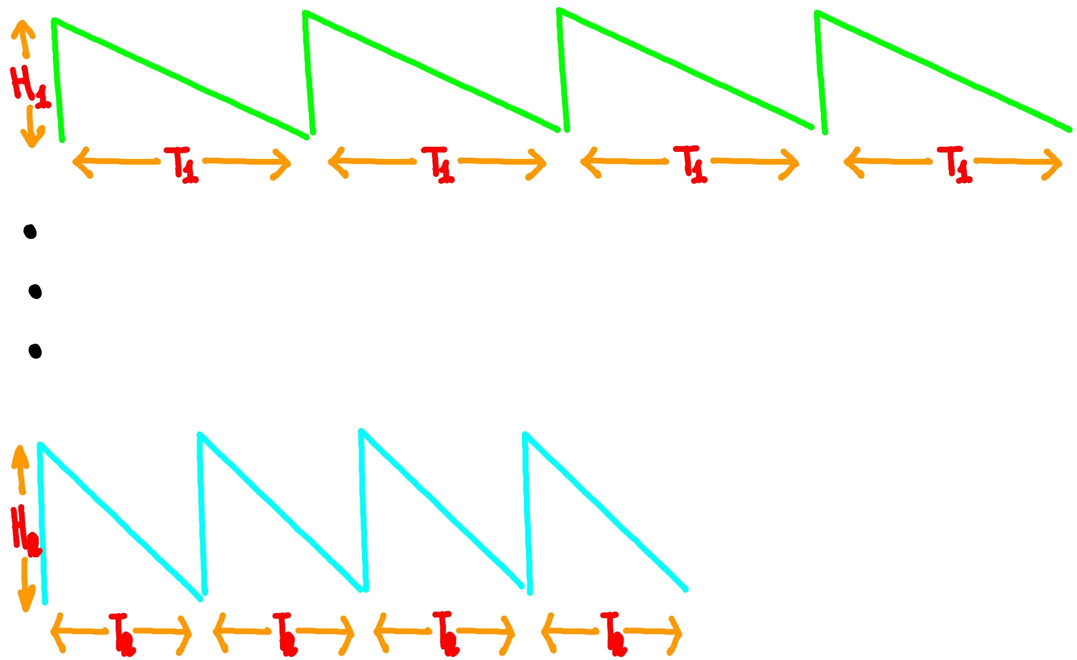
    $O(|I|^{O(1)} \cdot N^{O(N)})$ time via ILP in fixed dimension



# NESTED INSTANCES

- Structural assumption: Items can be sorted into $T_1 \leqslant \cdots \leqslant T_n$, such that $T_i | T_{i+1}$ for all $i = 1, \ldots, n-1$

Example: 2, 2, 2, 6, 18, 18, 72, ...



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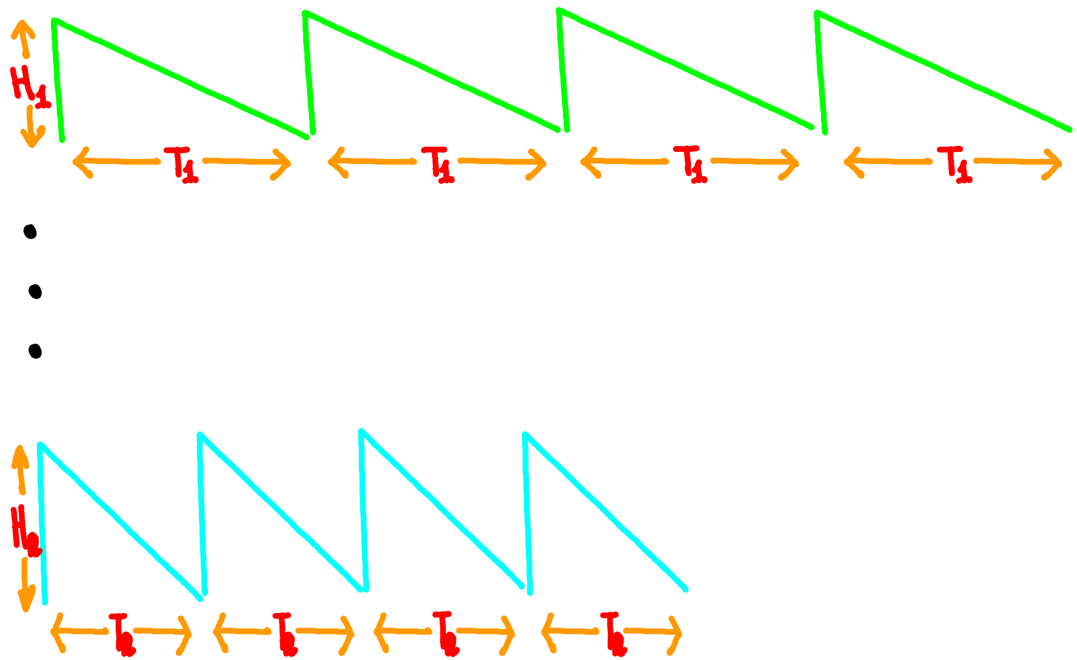

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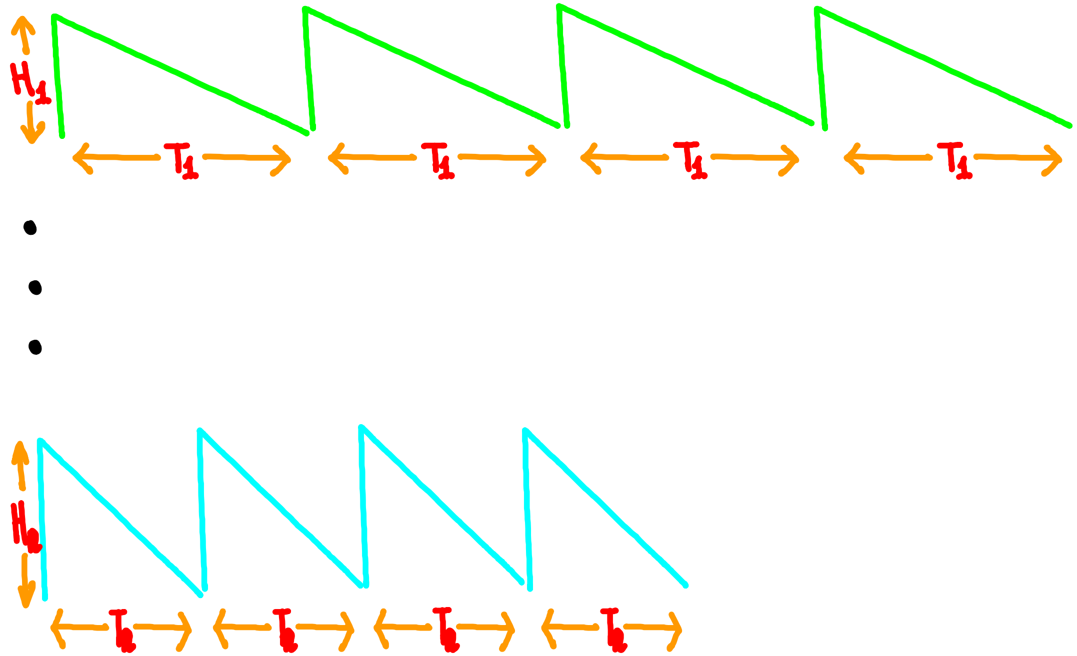


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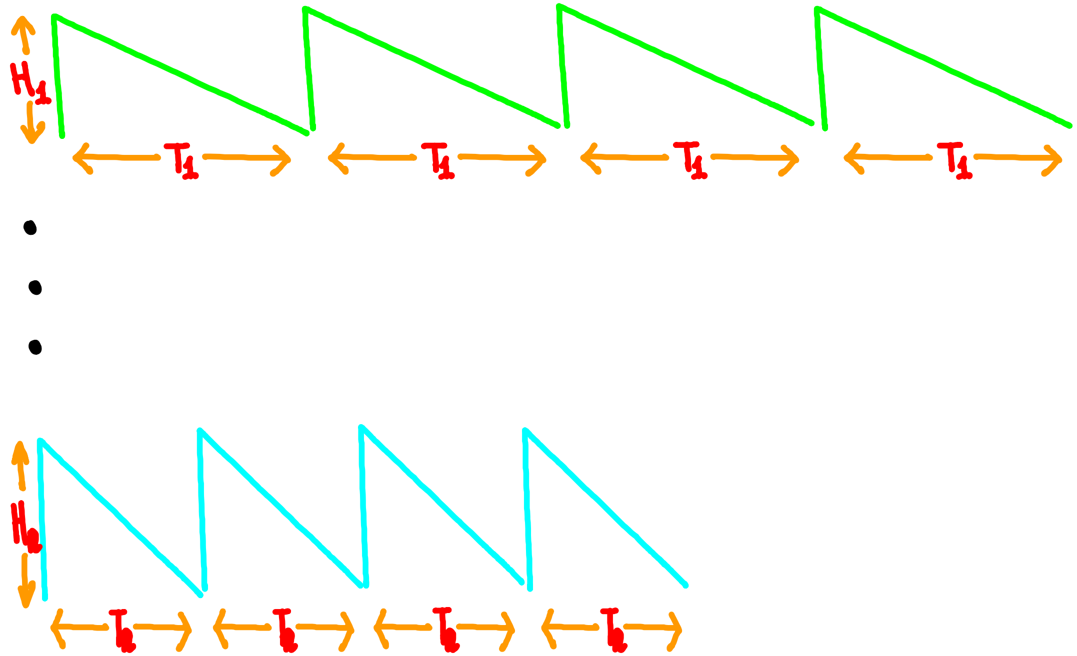


- OPEN QUESTION:

  Improved guarantee?

# NESTED INSTANCES

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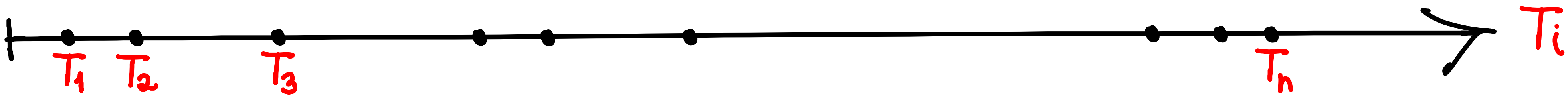

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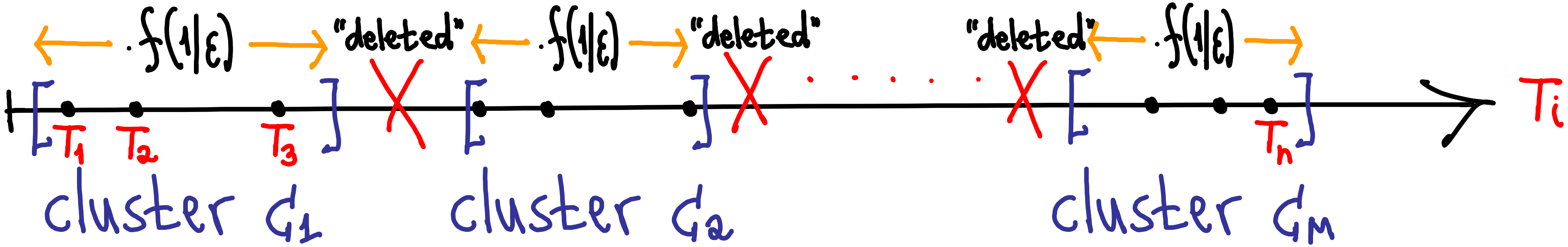

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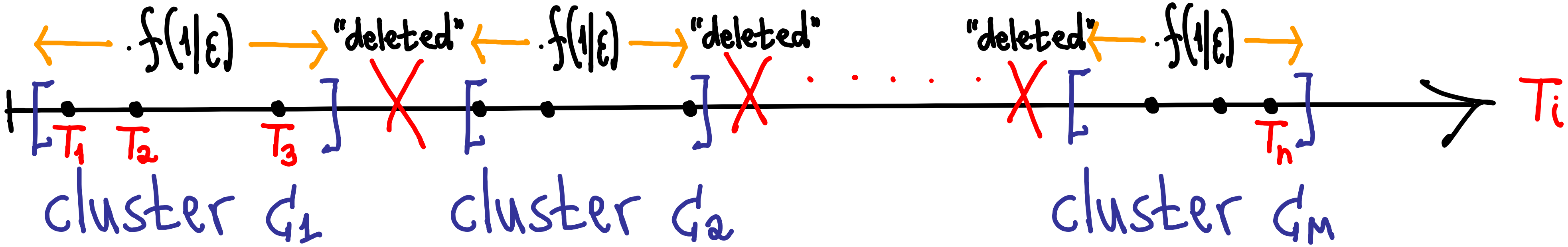


$$\sum_{i \text{ deleted}} H_i \leq \varepsilon \cdot \sum_{i \in [n]} H_i \Rightarrow \text{negligible}$$

average-space bound: $OPT \geq \frac{1}{2} \cdot \sum_{i \in [n]} H_i$

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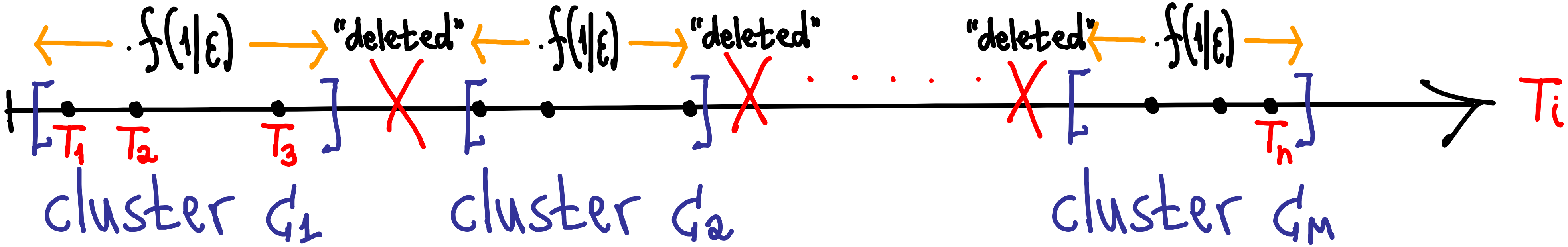


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nestedness

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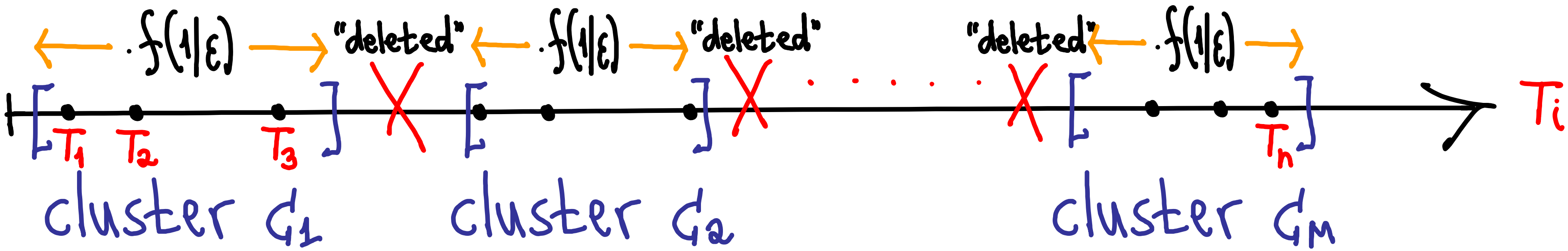


$\sum_{i \text{ deleted}} H_i \leq \varepsilon \cdot \sum_{i \in [n]} H_i \Rightarrow$ negligible

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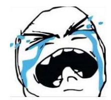
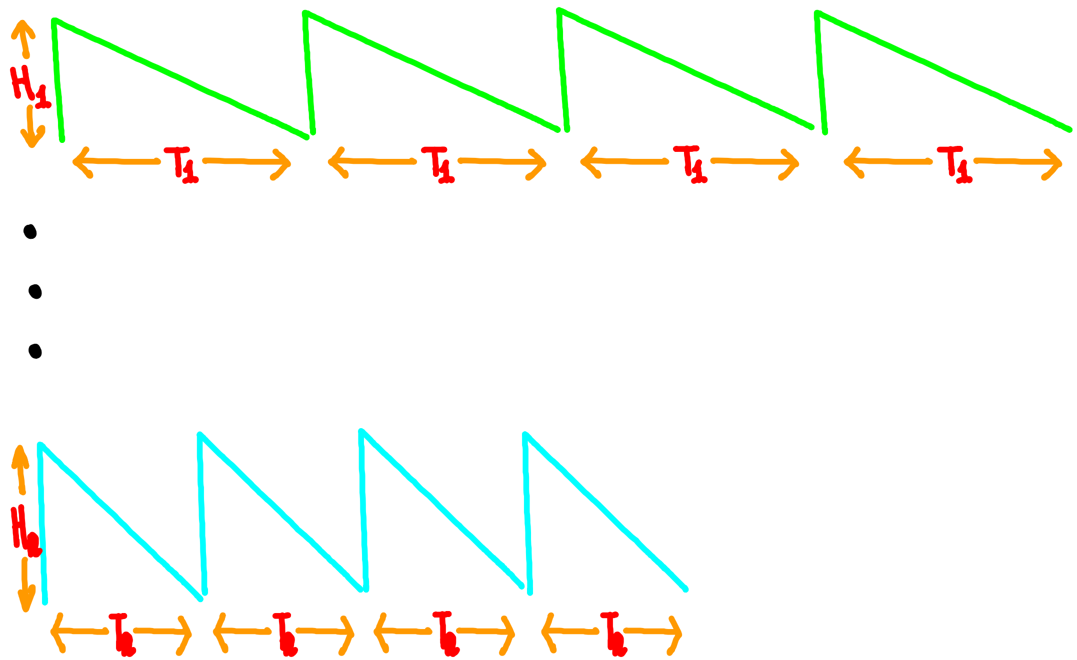
near-additivity: $OPT \geq (1-\varepsilon) \cdot \sum_{m \in [M]} OPT_m$ nestedness + 

with continuous shifts

# PAIRWISE-COPRIMES INSTANCES

- Structural assumption: $T_1, \dots, T_n$ pairwise coprime

  $T_i \neq T_j \Rightarrow \gcd(T_i, T_j) = 1$



# PAIRWISE-COPRIMES INSTANCES with continuous shifts

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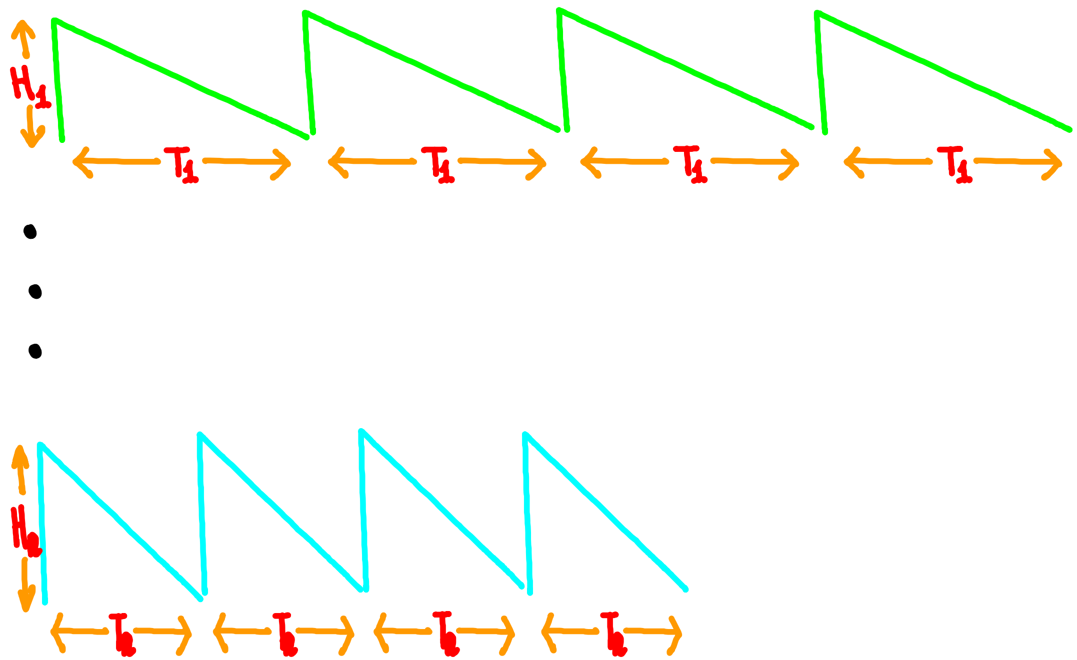

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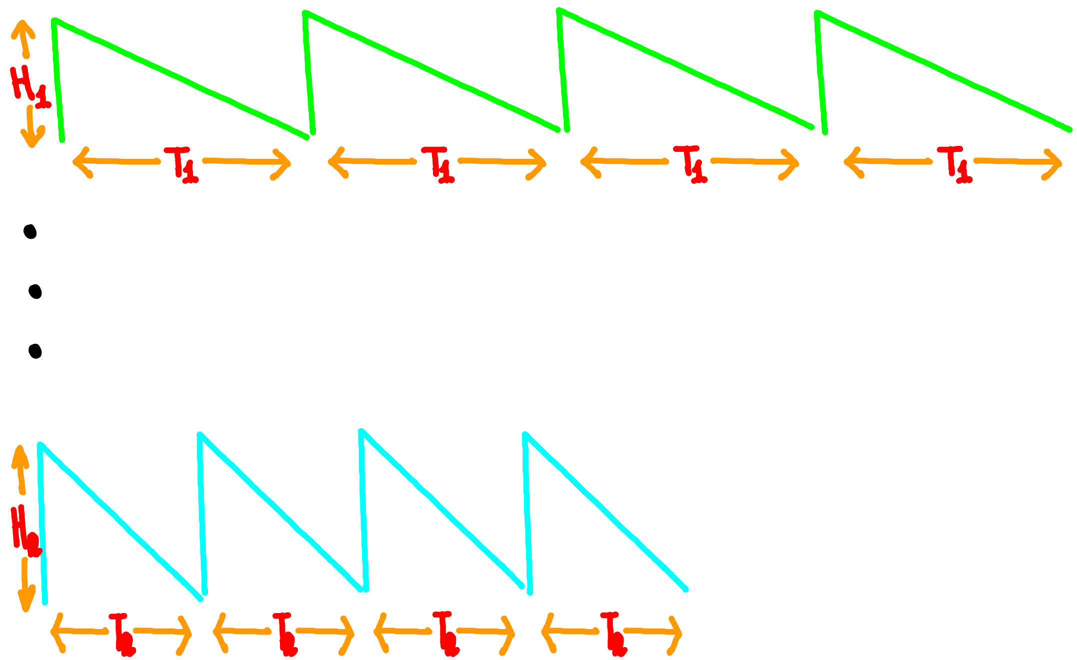

with continuous shifts

# PAIRWISE-COPRIMES INSTANCES

- Structural assumption: $T_1, \dots, T_n$ pairwise coprime
  $T_i \neq T_j \Rightarrow \gcd(T_i, T_j) = 1$
- Existing result: 1.4-approx [Hun, Sharafali, Teo '05]
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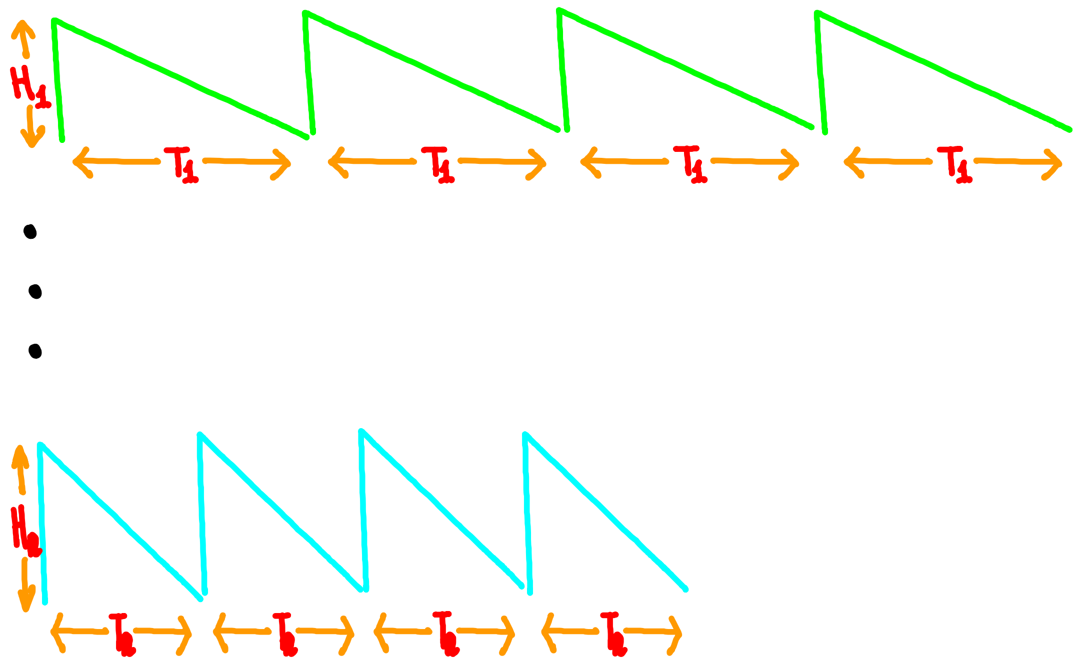
- THEOREM: $(1+\varepsilon)$-approx
  in $O(|I|^{O(1)} \cdot \tau_H(\frac{1}{\varepsilon}, \frac{1}{\varepsilon}))$ time



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$$\left.4^{4^{4^{\cdot^{\cdot^{\cdot^{4^{1/\varepsilon}}}}}}}\right\} 1/\varepsilon \text{ times}$$

worst $(1+\varepsilon)$-approx ever?

(without Regularity Lemma)

# OUTLINE

MODEL FORMULATION

IMPOSSIBILITY RESULTS

OLD/NEW APPROXIMATIONS

CONCLUDING REMARKS

# REMARKS AND OPEN QUESTIONS

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- Further details...

arXiv > cs > arXiv:2506.10339

**Computer Science > Data Structures and Algorithms**

*[Submitted on 12 Jun 2025]*

**New Approximation Guarantees for The Inventory Staggering Problem**

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THANKS FOR ATTENDING!