Online Integrated Production and Distribution Scheduling (Online IPDS)

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Outline

- IPDS Applications
- Online IPDS Problems & Challenges
- Two Specific Problems
- Survey Results

IPDS Applications

- Importance of joint scheduling of P & D
  - Production / order processing activities & distribution / delivery / transportation activities
  - Balance total cost & customer service
- Numerous applications

1. Manufacturers of make-to-order or time-sensitive products

   -- Time urgency ➔ speedy delivery, careful scheduling necessary

   -- Delivery soon after production ➔ coordination of the two functions is key

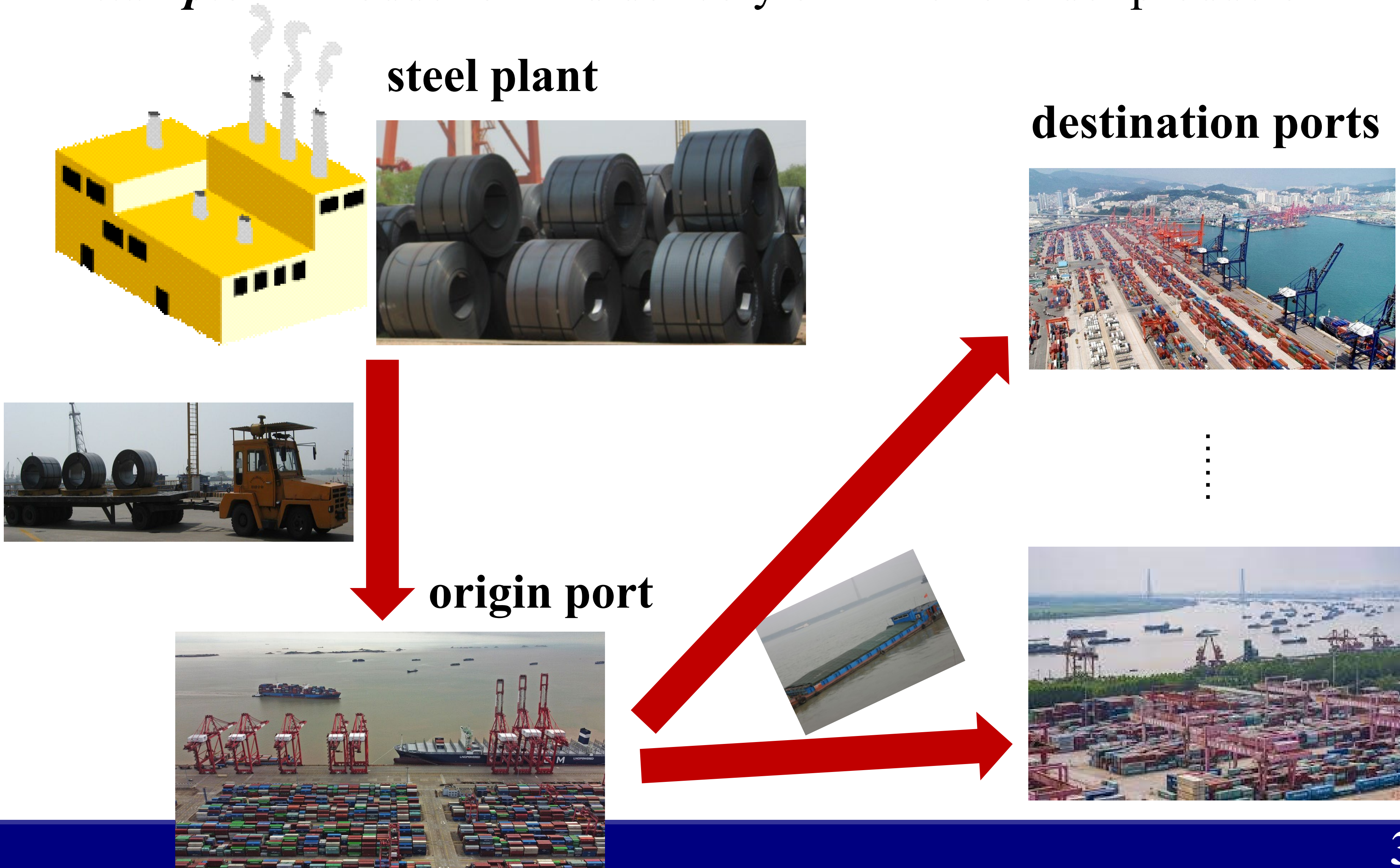
2. Online retailers

   -- Order fulfillment operations – order processing & last mile delivery

IPDS Applications

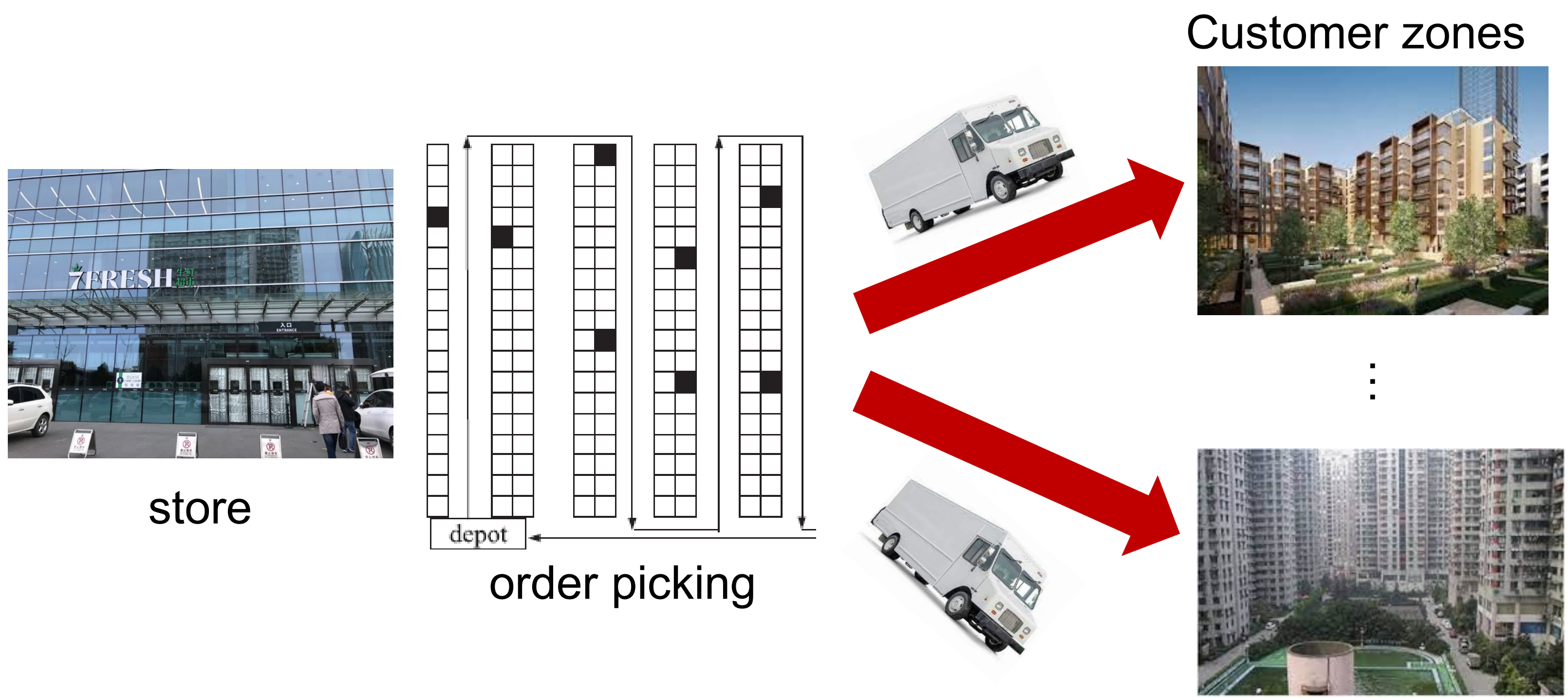
Tang, Li and Chen (2019)

- ***Example 1***: Production and delivery of make-to-order products



IPDS Applications

➢ ***Example 2*: Fulfillment of online orders**



Objective: Minimize sum of total transportation cost and max delivery lead time

Zhang, Wang and Huang (2018): a single order picker, multiple customer zones
Zhang, Liu, Tang and Li (2019): multiple pickers, a single customer zone
Zhang, Zhang and Zhang (2021): involve vehicle routing

IPDS Problems

- Two type of IPD scheduling (IPDS) problems
  - Offline problems
    - All problem parameters are known in advance (deterministically or probabilistically)
    - Decisions are made once only
    - *Most existing problems are offline problems*
  - Online problems
    - Most problem parameters are not known in advance / become known only after orders arrive
    - New decisions are made whenever a new order arrives
    - *Rapidly growing interest in online IPDS problems*

IPDS Problems

- Why making online (real-time) decisions?
  - Orders arrive randomly over time with little predictability
  - Customers expect a short delivery lead time

Online IPDS Problems

- Generally defined
  - Orders $N = \{1, 2, \ldots, \}$ from one or more customers $K=\{1,2,\ldots, \}$ arrive randomly over time.
  - Each order's arrival time, processing time, customer identity, etc. are not known until it arrives.
  - Orders are first processed in a production facility with one or more production / processing lines / machines
  - Completed orders are delivered to their customers
  - Information about delivery vehicles (number, capacity) is all known in advance. In most cases, transportation time and cost information is also all known in advance

Online IPDS Problems

- Key elements in an online IPDS problem
  - **Number of production lines**
  - **Number of customer locations**
  - **Number and capacity of delivery vehicles**
  - **Delivery Method:**
    - individually and immediately;
    - batch delivery to a single customer;
    - batch delivery to multiple customers – *direct shipping* (one customer per trip), or *routing* (multiple customers per trip)
  - **Objective:**
    - minimize a time-based performance measure (e.g., average lead time, max lead time), total transportation cost, etc.
    - maximize total revenue of covered orders, transportation capacity utilization, etc.

Online Algorithms

- Decisions are made piece by piece in a serial fashion, in the order of the input that is fed to the algorithm

- **Competitive ratio**

  measures the worst-case performance of an online algorithm

$$R^{\mathrm{A}} = \sup_{I \in \mathcal{I}} \{ Z^{\mathrm{A}}(I) / Z^{*}(I) \}$$

Competitive ratio of online algorithm A

Objective value of the solution generated by alg A for instance I

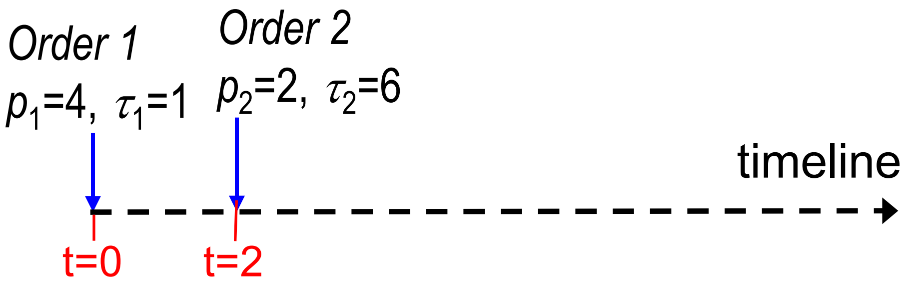
Optimal objective value of the offline version of the instance

Online Algorithms

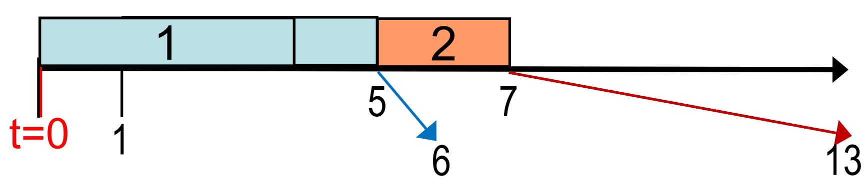
- A good waiting strategy is the key to performance
  - Waiting leads to more information
  - Waiting wastes production / delivery capacity
  - A well-balanced waiting strategy is needed

Online Algorithms

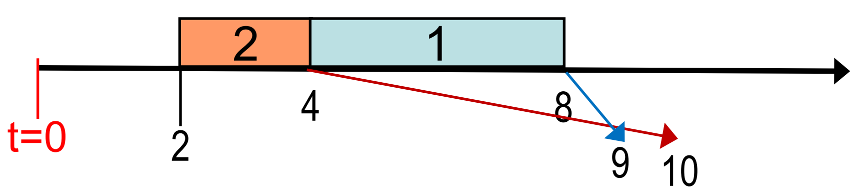
- *Example 1:*
  - Each incoming order belongs to a different customer;
  - A single production line;
  - **Individual and immediate delivery** upon completion; infinite number of delivery vehicles
  - Objective: minimize the time when all orders are delivered (maximum delivery time, $D_{\max}$)



Should have waited longer!



$D_1 = 6$; $D_2 = 13$ ➔ $D_{max} = 13$



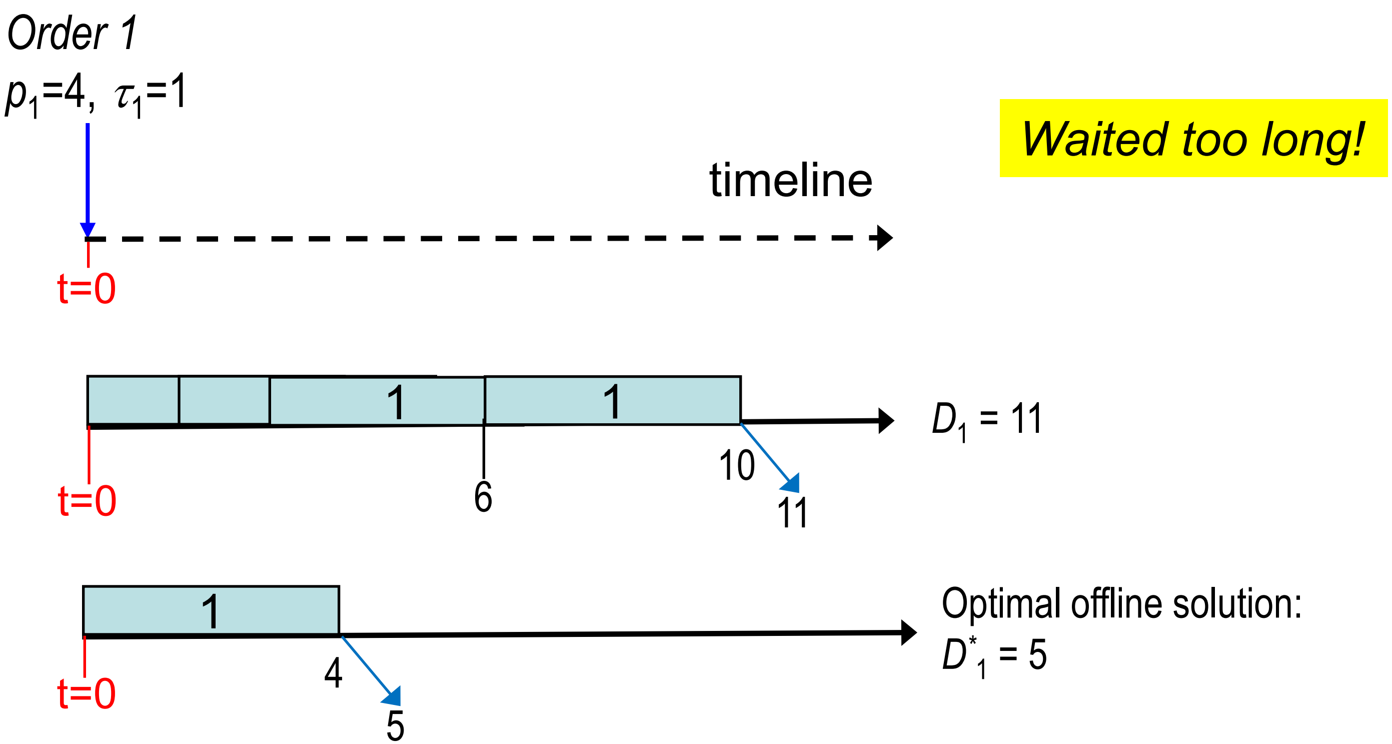
Optimal offline solution:

$D_2 = 10$; $D_1 = 9$ ➔ $D^*_{max} = 10$

$Z^A(I) / Z^*(I) = 13 / 10 = 1.3$

Online Algorithms

- *Example 1:*
  another instance

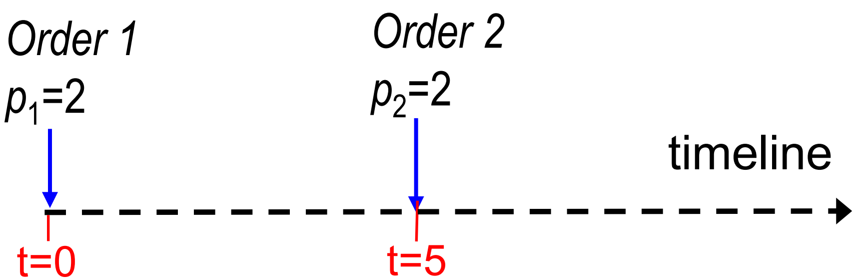


$Z^A(I) / Z^*(I) = 11/5 = 2.2$

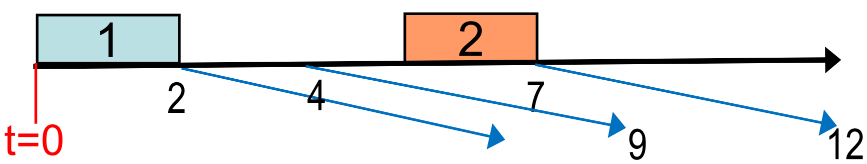
Online Algorithms

➢ *Example 2:*

- All incoming orders belong to the same customer – with delivery time 5;
- A single production line;
- **Batch delivery** (size 2); infinite number of delivery vehicles;
- Objective: minimize weighted sum of total delivery cost and average delivery lead time



Should have waited longer!

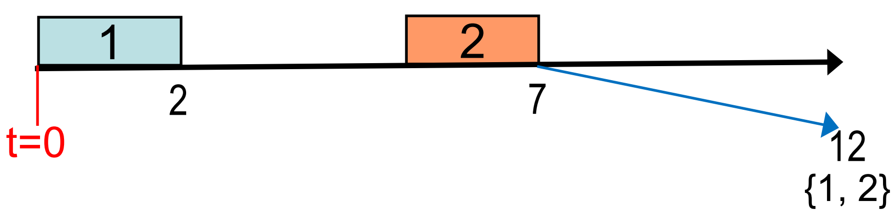


2 shipments

$D_1 = 9; \quad D_2 = 12$

➔ *avg delivery lead time* $\sum D_j / 2 = 21 / 2 = 10.5$

➔ *obj value* $= 2w + 10.5(1-w) = 10.5 – 8.5w$



1 shipment

$D_1 = D_2 = 12$

➔ *avg delivery lead time* $\sum D_j / 2 = 24 /2 = 12$

➔ *obj value* $= w + 12(1-w) = 12 - 11w$

$\mathbf{Z^A(I) / Z^*(I) = (10.5\text{-}8.5w) / (12\text{-}11w)}$

Problem: $1|online, r_j|V(v,c)|1|D_{\max} + TC$

- A single machine;
- Orders arrive randomly over time; they belong to a single customer;
- v delivery vehicles available, each with a capacity of c orders per shipment;
- Find a joint order processing and delivery schedule to minimize the time when all the orders are delivered ($D_{\max}$) and the total transportation cost (TC)

τ = one-way transportation time between the processing facility to the customer site;

f = transportation cost of each shipment;

$D_{\max} = \max\{D_j | j \in N\}$;

TC = the number of shipments used times f.

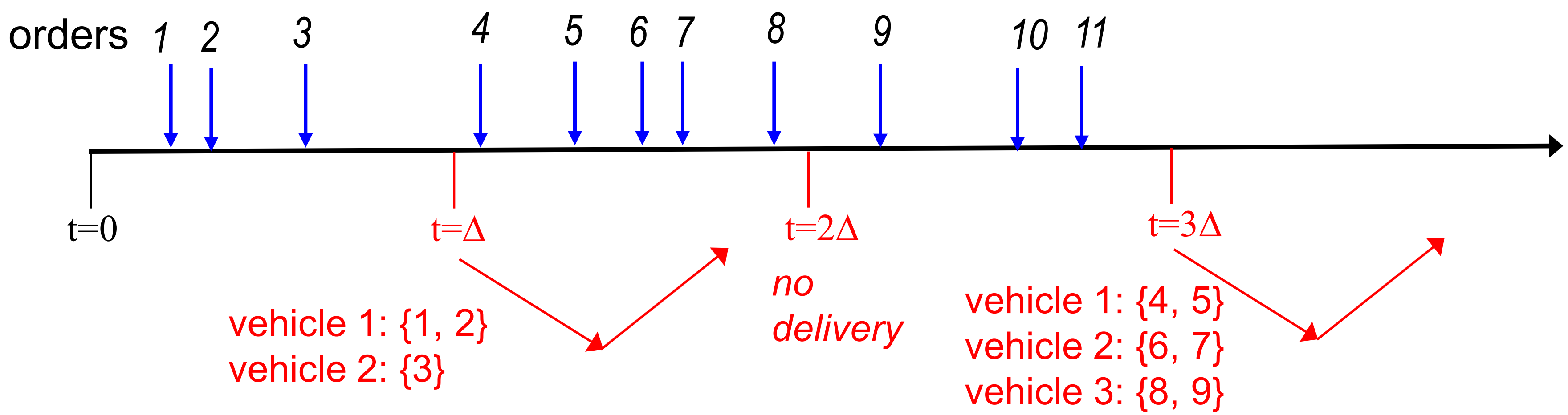
Algorithm

Step 1: In the production stage, whenever the machine is available, process any order that has arrived but has not been processed.

Step 2: In the delivery stage, consider the time points $l\Delta$, for $l = 1, 2, \ldots$, where $\Delta = \max\{2\tau, f\}$. At any time point $t = l\Delta$, if there is no order being processed, then deliver as many orders as possible using the v vehicles; otherwise, do not deliver any order.

Chen (2025):
competitive ratio 3.666…

Ex: $v = 3$, $c = 2$



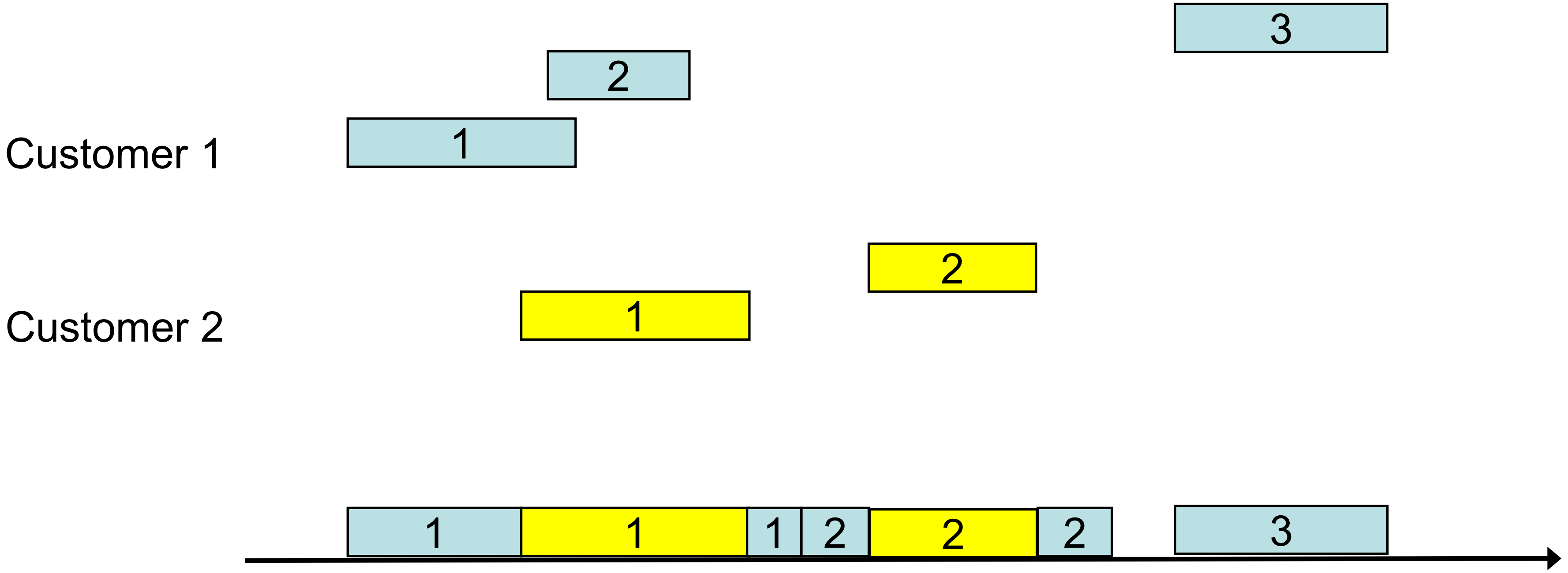
Problem: $1|online, r_j, pmtn|V(\infty, c), direct|k|D_{\max} + TC$

- A single machine;
- Orders arrive randomly over time; they belong to *k* customers;
  order processing can be preempted if necessary;
- Unlimited number of delivery vehicles available, each with a capacity of *c* orders;
- Direct shipping method is used;
- Find a joint order processing and delivery schedule to minimize the time when all the orders are delivered ($D_{\max}$) and the total transportation cost (TC)

For $i = 1, ..., k$,
τ_i = one-way transportation time between the processing facility to customer i;
f_i = transportation cost of each shipment to customer i.

Algorithm

LTT Rule: At each time instant whenever the machine becomes available or when a new order arrives, process the order with the longest transportation time among all the orders that have arrived but have not been completed, including those that have been partially processed.



Algorithm

Step 1: At the production stage, process the arriving orders by the LTT rule.

Step 2: At the delivery stage, for each customer i, at each time point $t_l^i = l(\sqrt{k} f_i)$, for $l = 1, 2, \ldots$, if all the orders from customer i that have arrived before this time have completed processing, then deliver them all to customer i using a minimum number of vehicles. Otherwise, do not deliver any orders from customer i.

Chen and Hall (2022):
competitive ratio: $\sqrt{k} + 1$

Existing Literature

- Research on IPDS problems:
  - Relatively recent
  - Most existing results appeared in the last 20 years
  - One of few SCM problem areas that are still growing

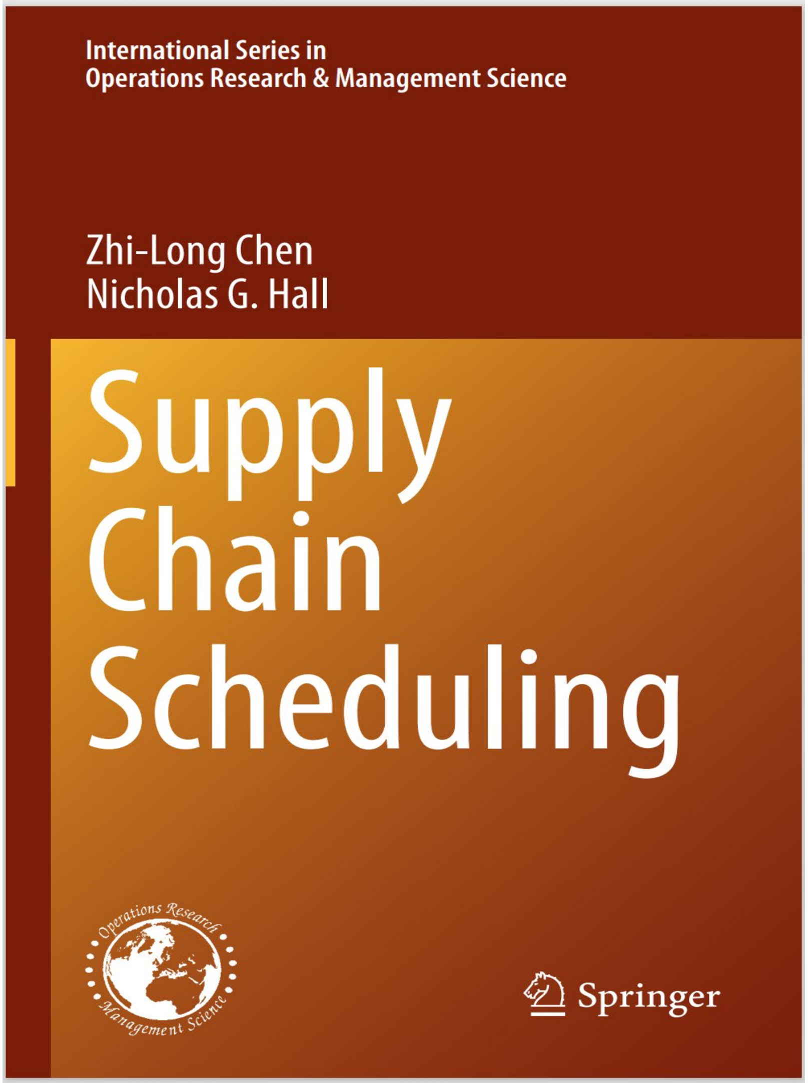
Survey Papers:

Offline IPDS Problems: Chen (2010), Wang et al. (2015), Moons et al. (2017)

Online IPDS Problems: Chen (2025)

Chen, Z.-L., 2025, Online Integrated Production and Distribution Scheduling: Review and Extensions, *INFORMS J. Computing, 37(2), 360-380*

Chen and Hall (2022)



Chapter 3 – Offline IPDS
Chapter 4 – Online IPDS

Survey on Online IPDS

Chen (2025) survey:

- Individual and immediate delivery
  - All existing problems involve unlimited number of delivery vehicles, and the objective function of $D_{\max}$
- Batch delivery to a single customer
  - All existing problems involve a single delivery vehicle, or unlimited number of delivery vehicles
  - Most involve a single production line
  - All involve objective function $D_{\max}$ or ΣD_j (+ TC)
- Batch delivery to multiple customers by direct shipping
  - All existing problems involve unlimited number of delivery vehicles, and a single production line
  - All involve objective function $D_{\max}$ or ΣD_j (+ TC)

Topics for Future Research

- Problems with multiple but limited number of delivery vehicles
- Problems with other objective functions
- Problems with packing considerations
- Problems in a semi-online environment

Thank you!

Questions?

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