DESTRUCTIVE AND CONSTRUCTIVE BOUNDS FOR THE m-MACHINE SCHEDULING PROBLEM

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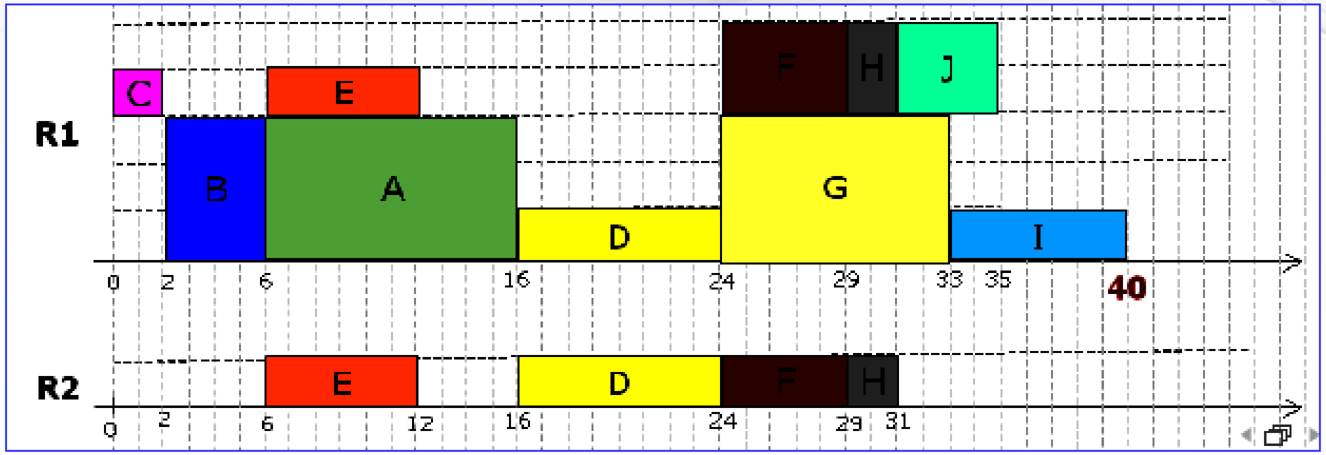

INTRODUCTION

- RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM (RCPSP)
- DECOMPOSITION INTO CUMULATIVE SCHEDULING PROBLEMS (CuSP) CONNECTED WITH THE PRECEDENCE GRAPH
- THE CuSP, THE m-MACHINE SCHEDULING PROBLEM (Carlier 1987, EJOR) (Haouari et al. 2007, JOS)
- CONSTRUCTIVE AND DESTRUCTIVE BOUNDS (Brucker 1990)
- ENERGETIC CONSTRUCTIVE BOUNDS

THE CUMULATIVE SCHEDULING PROBLEM (CuSP)

- m-MACHINE OPTIMISATION
  - Schedule n non preemptive tasks in a minimal makespan
  - Each task i has:
    - a release date r_i,
    - a processing time p_i
    - a tail q_i.
  - It requires c_i=1 machine during all its processing ($m = C$)

- m-MACHINE DECISION (C_{max}) **(constraint programming)**
  - A value C_{max} is chosen
  - In the m-machine decision, we replace tails by deadlines ($d_i(C_{max}) = C_{max} - q_i$)
  - Each task i has to be scheduled within the interval $[r_i, d_i]$

- THE CUMULATIVE SCHEDULING PROBLEM (CuSP):
  - A task can need more than one machine:
    - c_i is no more necessarily equal to 1

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PART 1 - DESTRUCTIVE BOUNDS

THREE CHECKERS

- $EB(\alpha, \delta)$: Energetic Balance of an interval $[\alpha, \delta]$

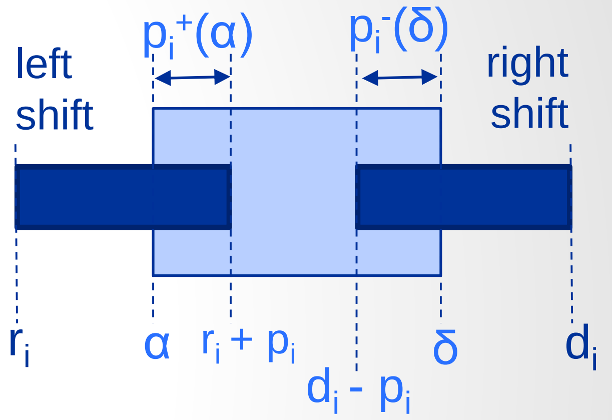
- Energetic Balance of all intervals
  - $EB = \min(EB(\alpha, \delta))$

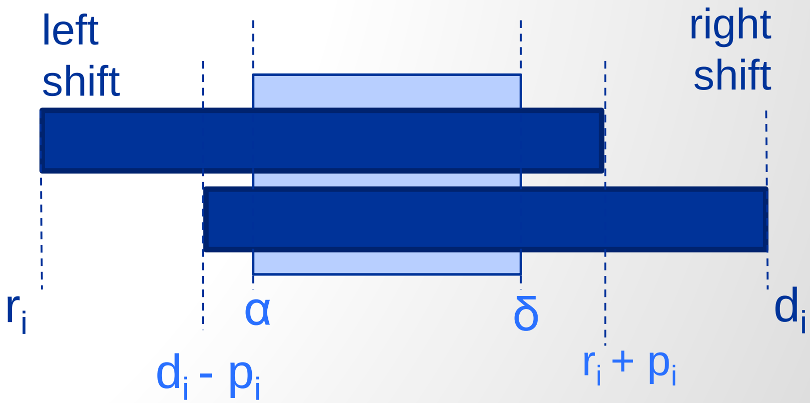
- Energetic Reasoning (ER) (Erschler and Lopez)
  - If $EB < 0$, the instance is infeasible and $C_{max} + 1$ is a valid lower bound.

ENERGETIC REASONING: A DESTRUCTIVE BOUND

- **TAILS ARE REPLACED BY DEADLINES**
- Energetic Reasoning (ER) (Erschler and Lopez, Baptiste, Le Pape and Nuijten)
- Given a time interval $[\alpha, \delta]$
  - Let $p_i^+(\alpha)$ the length of time during which task i after α if it is left-shifted
  - Let $p_i^-(\delta)$ the length of time during which task i before δ if it is right-shifted
  - $W_i(\alpha,\delta) = c_i \times \min(p_i^+(\alpha), p_i^-(\delta), \delta - \alpha)$
- The total energy over the time interval $[\alpha, \delta]$ is defined by $W(\alpha,\delta) = \sum_{i=1}^{n} W_i(\alpha,\delta)$.
- $EB(\alpha,\delta) = C(\delta - \alpha) - W(\alpha,\delta)$ and $EB = \min\big(EB(\alpha,\delta)\big)$

Clearly, if $EB < 0$, the instance is infeasible. Otherwise it could be feasible.





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THE FAMILY OF INTERVALS

THE FAMILY OF INTERVALS $[\alpha, \delta]$: (the pinning points)

- Family of intervals Ω_1
  - $\alpha \in \{r_i, r_i + p_i, d_i - p_i(crossing\ task) \mid i \in \{1, \dots, n\}\}$
  - $\delta \in \{d_i, r_i + p_i(crossing\ task) \quad \mid i \in \{1, \dots, n\}\}$
- Family of intervals Ω_2
  - $\alpha \in \{r_i, d_i - p_i(crossing\ task) \mid i \in \{1, \dots, n\}\}$
  - $\delta \in \{r_k + d_k - \alpha \quad \mid k\ balancing\ \boldsymbol{equilibrium}\ task\}$
    - where k is a function of α
- Family of intervals Ω_3
  - $\delta \in \{d_i, r_i + p_i(crossing\ task) \quad \mid i \in \{1, \dots, n\}\}$
  - $\alpha \in \{r_k + d_k - \delta \quad \mid k\ balancing\ \boldsymbol{equilibrium}\ task\}$
    - where k is a function of δ

Total number of intervals : $n^2 + 4nm + m^2$

ENERGETIC REASONING: LITERATURE REVIEW

- **Baptiste, Le Pape and Nuijten (1999) proposed a quadratic checker. They also derived a cubic algorithm for computing heads and tails adjustments.**

- Challenges of ulterior researches:
  - **Can we do better than quadratic complexity for checker?**
  - **Can we do better than cubic algorithms for adjustments?**

- Brief history of adjustment improvements:
  - $O(n^2 \log n)$ (Bonifas 2018, Tesch 2018, Ouellet Quimper 2018)
  - $\boldsymbol{O(n^2)}$**: OUR ADJUSTMENTS ALGORITHM (Incremental evaluation and Cooling box: hare, tortoises etc.)**
    - Carlier, J., Pinson, E., Sahli, A. and, Jouglet, A. (2020). An O(n^2) algorithm for time-bound adjustments for the cumulative scheduling probem. European Journal of Operational Research, vol 286(2), 468-476.
    - Carlier, J., Jouglet, A Pinson, E., Sahli, A. (2020). A new data structure for some scheduling problems: **the cooling box**. JOCO.

- We have evaluated the incremental addition of the constraint ri= α to the evaluation of energy in the double loop of Baptiste et al. The method is made efficient by using adapted data structure including a new one: the cooling box.

ENERGETIC REASONING: A DESTRUCTIVE BOUND

THE QUADRATIC CHECKER OF BAPTISTE, LE PAPE AND NUIJTEN

- Let us define the sets:

$O_1(i) = \{r_i, r_i + p_i, d_i - p_i\}, \forall i \in \{1, \dots, n\}$ $\quad O_1 = \cup_{i \in \{1,\dots,n\}} O_1(i)$

$O_2(i) = \{r_i + p_i, d_i - p_i, d_i\}, \forall i \in \{1, \dots, n\}$ $\quad O_2 = \cup_{i \in \{1,\dots,n\}} O_2(i)$

$O_t(i) = \{r_i + d_i - t\}, \forall i \in \{1, \dots, n\}$ $\quad O_t = \cup_{i \in \{1,\dots,n\}} O_t(i)$

Proposition 1 [Baptiste et al. 2001]: It is sufficient to check intervals [α,δ] in $\Omega = \Omega_A \cup \Omega_B \cup \Omega_C$ with three families:

$$\Omega_A = \{[\alpha, \delta] \mid \alpha \in O_1, \delta \in O_2, \alpha < \delta\}$$
$$\Omega_B = \{[\alpha, \delta] \mid \alpha \in O_1, \delta \in O_\alpha, \alpha < \delta\}$$
$$\Omega_C = \{[\alpha, \delta] \mid \delta \in O_2, \alpha \in O_\delta, \alpha < \delta\}$$

- The number of such intervals is equal to $15n^2$
- Improved by Derrien and Petit to $3n^2$ (us: nearly n square)
- **Thanks to two double loops on α and δ and incremental evaluations. They also derived a cubic algorithm for computing heads and tails adjustments.**

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ENERGETIC REASONING: A DESTRUCTIVE BOUND

- FORMULA (Checker)

$$p_i^+(\alpha) = \max(0, \min(p_i, r_i + p_i - \alpha))$$
$$p_i^-(\delta) = \max(0, \min(p_i, \delta - d_i + p_i))$$
$$W_i(\alpha, \delta) = \min(p_i^+(\alpha), p_i^-(\delta), \delta - \alpha)$$

- INTERVALS FAMILIES (Baptiste et al. Checker)

$$\Omega_A = \left\{(\alpha, \delta) \middle| \begin{array}{l} \boldsymbol{\alpha\ of\ the\ form}:\ r_i\ or\ d_i - p_i\ or\ r_i + p_i \\ \boldsymbol{\delta\ of\ the\ form}:\ d_j\ or\ d_j - p_j\ or\ r_j + p_j \end{array}\right\}$$

$$\Omega_B = \left\{(\alpha, \delta) \middle| \begin{array}{l} \boldsymbol{\alpha\ of\ the\ form}:\ r_i\ or\ d_i - p_i \\ \boldsymbol{\delta\ of\ the\ form}:\ r_j + d_j - \alpha \end{array}\right\}$$

$$\Omega_C = \{symmetrical\ case\ of\ \Omega_B\}$$

- FORMULA AND INTERVALS FAMILIES (BOUNDS)

$$p_i^+(\alpha) = \max(0, \min(p_i, r_i + p_i - \alpha))$$
$$p_i^-(\delta) = \max(0, \min(p_i, \delta - d_i + p_i)), \qquad \delta = C_{max} - \gamma$$
$$W_i(\alpha, \delta) = \min(p_i^+(\alpha), p_i^-(\delta), C_{max} - \gamma - \alpha)$$

ENERGETIC REASONING: A DESTRUCTIVE BOUND

THE CHECKER OF OUELLET AND QUIMPER

- Ouellet and Quimper have proposed recently a $O(n\log^2 n)$ checker and an $O(n^2\log n)$ algorithm for ajustments (2018).

- It answered to the challenge of Baptiste et al.

- They build a very clever algorithm based on range trees for computing the energy of an interval in $O(\log n)$ (tools issued from algorithmic geometry) **PRETREATMENT WITH RANGE TREES**

- They prove the following fundamental property: **PARADIGM CHANGEMENT**
  - The matrix of energy interval is a Monge Matrix.

- The lines of the matrix are associated with the values of α and the column with the values of δ.

- Two difficulties :
  - The Monge Matrix is a Partial Monge Matrix
  - There are a quadratic number of lines and of columns.

- They overcome these difficulties by a clever algorithm.

ENERGETIC REASONING: A DESTRUCTIVE BOUND

THE CHECKER OF CARLIER, SAHLI, JOUGLET AND PINSON (IJPR 2021)

- At first we treat the second and third families of intervals by stating an **equilibrium property** associating with each value of alpha or delta a single interval.
- It permits to divide by n the number of these intervals in family 2 and family 3.
- We propose Algorithm 1 to compute all these specific intervals in $O(n \log n)$.
- Of course for the first family, the submatrix remains an inverse Monge matrix (So we cannot used directly the so-called SMAWK-algorithm which is linear).
- Note that each entry of the matrix is computed in O(log n) time using the method of (Ouellet and Quimper 2018).
- If for some row, the minimal value is strictly negative, then the considered instance is infeasible. The overall complexity of this Algorithm 2 is $O(\alpha(n)n \log n)$ ($\alpha(n)$ Ackermann coefficient).

Klawe, Maria, and Daniel Kleitman. 1990. An Almost Linear Time Algorithm for Generalized Matrix Searching. SIAM J. Discrete Math. 3: 81–97.

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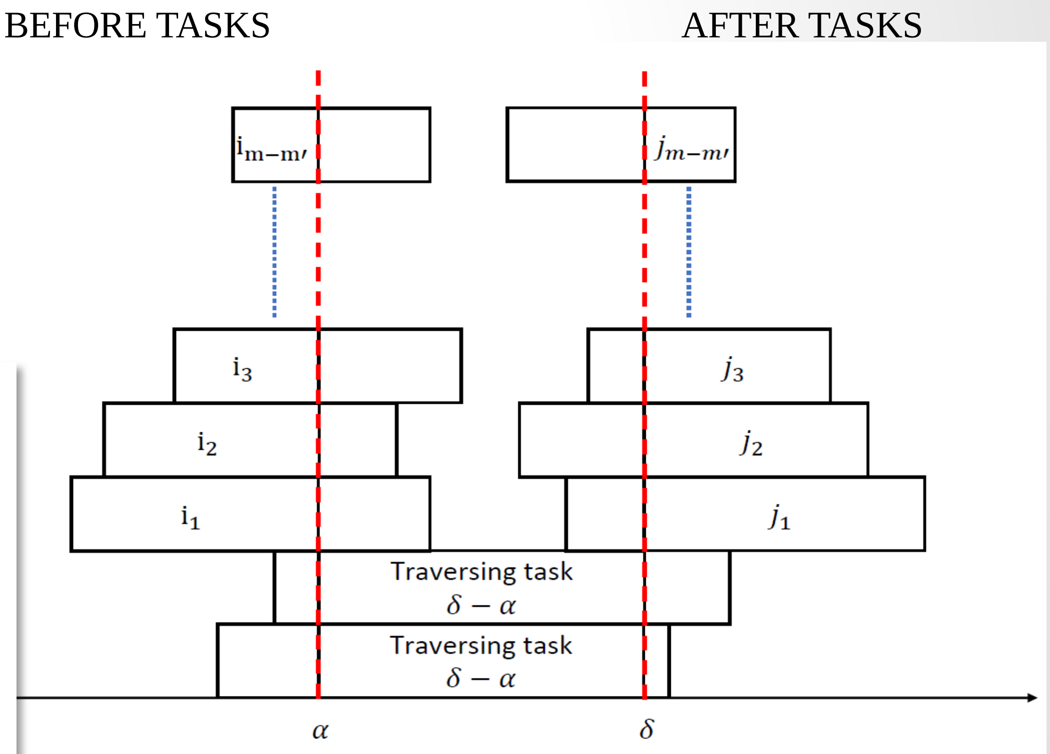
STRICLY NEGATIVE ENERGETIC BALANCE

- BEFORE TASKS:
  - $p_i^+(\alpha) \leq p_i^-(\delta)$
- AFTER TASKS:
  - $p_i^+(\alpha) \geq p_i^-(\delta)$
- BALANCING TASKS:
  - $p_i^+(\alpha) = p_i^-(\delta)$ and $\alpha + \delta = r_i + d_i$



$K(\alpha) = \{i \mid r_i \leq \alpha < r_i + p_i\}$

EQUILIBRIUM PROPERTY

Let us suppose that the minimal ENERGETIC BILAN of an interval $[\alpha, \delta]$ is strictly negative (« sursaturated interval »),

$$\begin{cases} \alpha \in \{r_i, d_i - p_i\} \quad and \\ \delta \in \{d_j, r_j + p_j, rj + d_j - \alpha\} \end{cases} \quad \text{or} \quad \begin{cases} \delta \in \{d_j, r_j + p_j\} \quad and \\ \alpha \in \{r_i, d_i - p_i, r_j + d_j - \delta\} \end{cases}$$

we have m BEFORE TASKS and m AFTER TASKS.

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STRICLY NEGATIVE ENERGETIC BALANCE

- Let $K(\alpha)$ be the set of tasks which meet α when they are left shifted. $\boldsymbol{K(\alpha) = \{i \mid r_i \leq \alpha < r_i + p_i\}}$
- Let $<_\alpha$ be a total strict total order between tasks:

$$i <_\alpha j \Leftrightarrow \begin{cases} rank(\alpha, i) < rank(\alpha, j) & or \\ rank(\alpha, i) = rank(\alpha, j) \text{ and } i < j \end{cases}$$

with:

$$rank(\alpha, i) = \begin{cases} 0 & if\ \alpha \geq d_i - p_i \\ r_i + d_i & if\ \alpha < d_i - p_i \end{cases}$$

- The set $K(\alpha)$ is ordered according to $<_\alpha$.
  - Let k and k' be the m^{th} and $(m+1)^{th}$ tasks of $K(\alpha)$ respectively (k' is supposed to exist):
  - Let $\delta_1 = r_k + d_k - \alpha$ and $\delta_2 = r_{k'} + d_{k'} - \alpha$

Critical interval proposition

There exists a critical interval such that δ is strictly larger than δ_1 and smaller or equal to δ_2.

- **This proposition permits to divide by n the number of intervals of families 2 and 3 of Baptiste et al.**
- δ_1 and δ_2 depends on α and δ, all of them can be computed by Algorithm 1 we elaborate.

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PART 2 - CONSTRUCTIVE BOUNDS

DESTRUCTIVE BOUNDS

- Baptiste, Le Pape and Nuijten: $O(n^2)$
- Ouellet and Quimper: $O(n \log^2 n)$
- Carlier, Sahli, Jouglet and Pinson: $O(\alpha(n) n \log n)$
- Practical complexity (function depends of n) are confirmed by computational results for any n
- The checker of Baptiste, Le Pape and Nuijten remains valuable because:
  - It brings more information (adjustments)

CONSTRUCTIVE BOUNDS

- **First alternative**:
  - Use a **checker** and apply a **dichotomic search**
  - It is not always good because the complexity is multiplied by $log(C_{max})$ so at least multiplied by $log\ n$
- **Second alternative**:
  - Characterize **mathematically the bound** and imagine other **nice algorithms**

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PART 2 - CONSTRUCTIVE BOUNDS

- $LB_1 = \max(r_i + p_i + q_i)$
  - Critical Path Bound
- LB_2 = a constructive time table bound
  - Algorithm 3: $O(n \log n)$
  - Degenerate case : the minimal intervals are of lengh 0
- LB_3= a constructive critical interval bound
  - Algorithm 4: $O(n^2)$
- LB_4= Jackson Pseudo Preemptive Schedule
- LB_5= the preemptive Schedule
  - imposed idle periods

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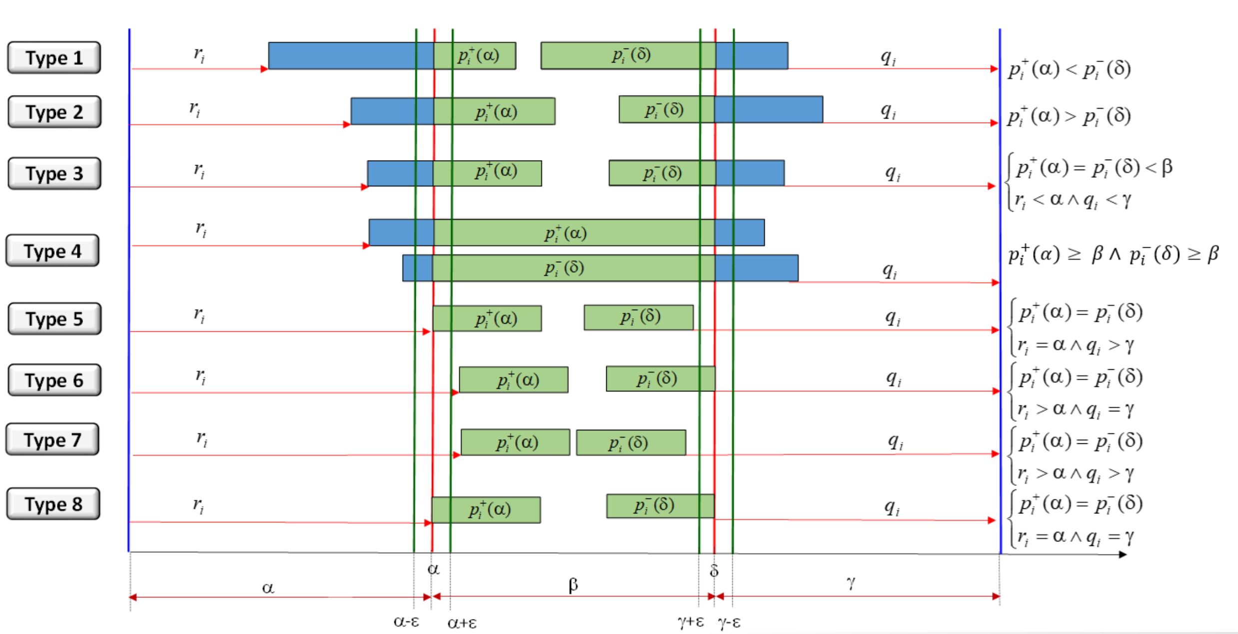
ENERGETIC REASONING: THREE CONSTRUCTIVE BOUNDS

- On this figure you can see 8 types of tasks for an interval and especially Type 4 which is the CROSSING TASK Type
  - See also the “parties obligatoires” of Lahrichi, RAIRO 1982
- We look for the smallest value $C_{max} = ER$ accepted by the checker
- **Equilibrium property**
- It appears a discontinuity due to crossing tasks so: $ER = \max(LB_2, LB_3)$
  - See: Carlier, Jouglet, Pinson and Sahli, a quadratic algorithm for computing the energetic bound, PMS 2021



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ENERGETIC REASONING: LB_2 - THE TIME TABLE BOUND

- **Adjusting the trial makespan to keep at most m crossing operations (called cumulative constraint) leads to the time table lower bound LB_2.**
  - Given a makespan C_{max} and a time instant t, a crossing operation satisfies $d_i - p_i \leq t < r_i + p_i$. Clearly, such an operation is always running in the interval $[t-1, t]$ for any non-preemptive schedule.
  - An immediate consequence is that if there are strictly more than m crossing operations at time t, then no non-preemptive schedule with a makespan less than or equal to C_{max} can exist.
  - This bound results from an adjustment of the trial makespan C_{max} ensuring that at any time instant t, there are at most m crossing operations, which can easily be tested by checking that there is no interval $[r_i + p_i - 1\,, r_i + p_i]$ in which $m+1$ operations are processed.
- This technique is well known, it is called time tabling.

Example: Consider the instance where m=2 machines and involving n=3 operations, each operation having a processing time equal to 1, a release date equal to 0, and a tail equal to 0. We have: $LB_2 = 2$

We have proposed an $O(n \log n)$ algorithm for computing LB_2

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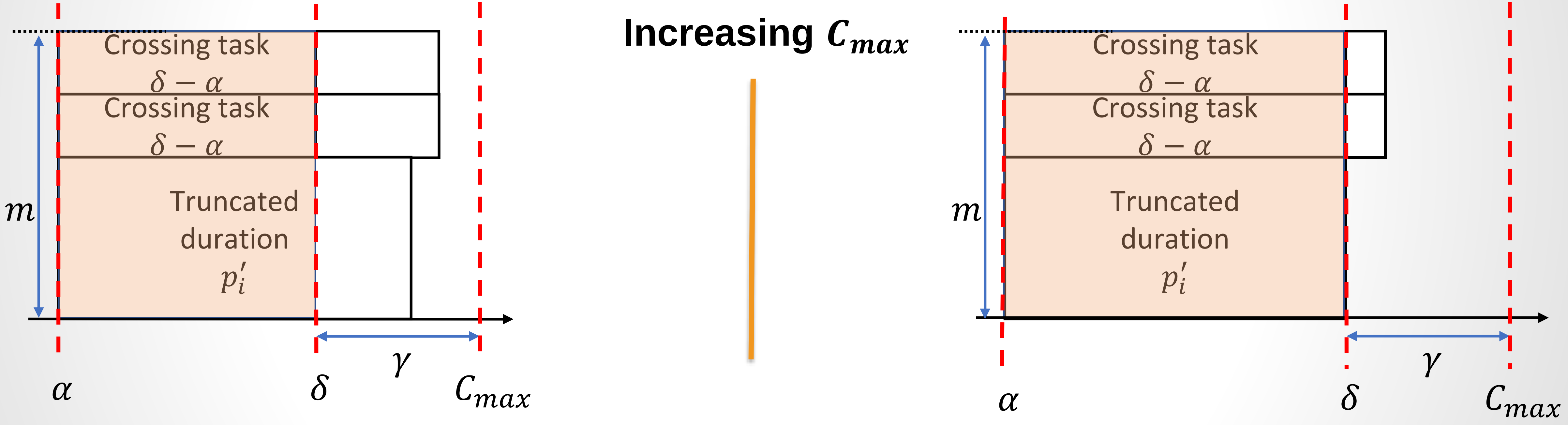
ENERGETIC REASONING: LB_3^{ER} - THE CRITICAL INTERVAL BOUND

An $O(n^2)$ algorithm



- ❖ The truncated duration: $\min(r_i + p_i - \alpha, q_i + p_i - \gamma, p_i, 0, \delta - \alpha)$
- ❖ **Double loop on α and γ**
- ❖ The constructive bound: $\boldsymbol{LB_3}$ is obtained when there exists a saturated interval (critical interval)

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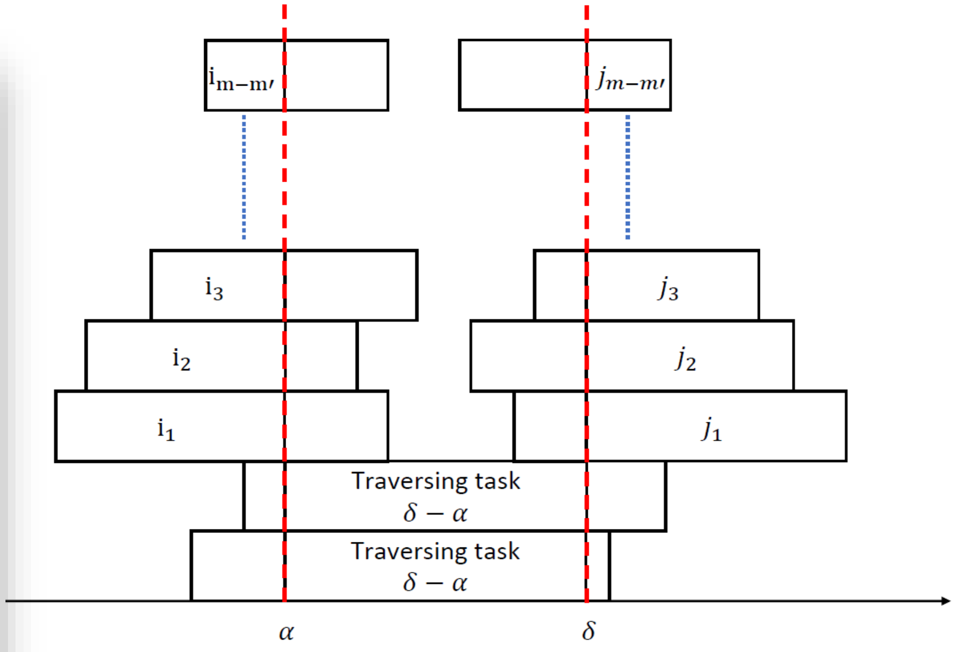
ENERGETIC REASONING: THE CRITICAL INTERVAL BOUND

Energy Theorem

Theorem 1 (Energy Theorem). *For a critical triplet $(\alpha^*, \beta^*, \gamma^*)$, we have:*

$$\alpha^*+\beta^*+\gamma^* = \frac{1}{\tilde{m}}\left[(r_{i_1}+r_{i_2}+\ldots+r_{i_{\tilde{m}}}) + \sum_{i\in \hat{J}(\alpha^*,\gamma^*)} p_i + (q_{j_1}+q_{j_2}+\ldots+q_{j_{\tilde{m}}})\right]$$

m' denoting the number of crossing operations, $\hat{J}(\alpha^*,\gamma^*) = J(\alpha^*,\gamma^*) - J_4(\alpha^*,\gamma^*)$ the subset of $\tilde{m} = m - m'$ non-crossing operations with a strictly positive energy on the time interval $[\alpha^*, \delta^*]$ with $\delta^* = C_{\max} - \gamma^*$, and where $\{i_1, i_2, \ldots, i_{\tilde{m}}\} = J_B$, $\{j_1, j_2, \ldots, j_{\tilde{m}}\} = J_A$, with $J_B \cap J_A = \emptyset$.



A critical interval

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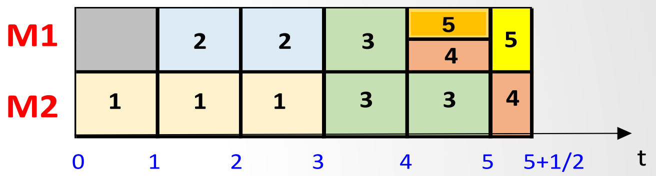
THE JPPS CONSTRUCTIVE BOUND

```
P(t)=∅ ; np(t)=m
While np(t)>0 do
  Compute PA and TA, the sets of non in-process partially (resp. totally) available operations with
  maximal priority
  If PA≠∅ or TA≠∅ then
    If |PA|+|TA| ≤np(t) then
      ∀i ∈PA, s_i(t) = 1; np(t) = np(t) − |PA|
      If |TA|>0 then
        ∀i ∈TA, s_i(t) = [np(t) − |PA|]/|TA|; np(t) = 0
      Endif
    Else
      ∀i ∈PA∪TA, s_i(t) = np(t)/|PA|+|TA|; np(t) = 0
    Endif
    P(t) = P(t)∪PA∪TA
  Else
    C(JPPS)=t
  Endif
Enddo
```

i	1	2	3	4	5
r_i	0	1	2	3	3
p_i	3	2	2	1	1
q_i	4	4	1	0	0



Theorem 2

$$LB^{JPPS} = C^{JPPS}_{max} = \max\left\{\max_{i\in I}(r_i + p_i + q_i), \max_{J\subseteq I, |J|\geqslant m} LB_2^{SB}(J)\right\}$$

J denoting a subset of operations of I with $|J| \geqslant m$, and $LB_2^{SB}(J)$ the quantity defined by:

$$LB_2^{SB}(J) = \frac{1}{m}(r_{i_1} + r_{i_2} + \dots + r_{i_m}) + \frac{1}{m}\sum_{j\in J} p_j + \frac{1}{m}(q_{j_1} + q_{j_2} + \dots + q_{j_m})$$

where $i_1, i_2, \dots, i_m$ (resp. $j_1, j_2, \dots, j_m$) denote the m first jobs in J rearranged in an ascending order of heads (resp. tails).

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THE PREEMPTIVE BOUND

- Intervals with idleness periods in some intermediary intervals which are necessary

Theorem 3

$$LB^{PB} = \frac{1}{m}\sum_{j\in J_a} r_j + \frac{1}{m}\left\{\sum_{j\in \bar{J}} p_j + \sum_{k\in \bar{K}} MH_k\right\} + \frac{1}{m}\sum_{j\in J_b} q_j$$

- Empirical results: the three bounds have most often the same value

- **Carlier, Pinson, Sahli et Jouglet 2020, Comparison of three lower bounds for the CusP (submitted)**

ER

$$LB^{ER} = \frac{1}{\tilde{m}}\left[(r_{i_1} + r_{i_2} + \ldots + r_{i_{\tilde{m}}}) + \sum_{i\in \hat{J}(\alpha^*,\gamma^*)} p_i + (q_{j_1} + q_{j_2} + \ldots + q_{j_{\tilde{m}}})\right]$$

$\tilde{m}$: number of non-crossing operations

$$\hat{J}(\alpha^*,\gamma^*) = J(\alpha^*,\gamma^*) - J_4(\alpha^*,\gamma^*)$$

$\{i_1,i_2,\ldots,i_{\tilde{m}}\} = J_B$, $\{j_1,j_2,\ldots,j_{\tilde{m}}\} = J_A$ with $J_B \cap J_A = \emptyset$.

JPPS

$$LB^{JPPS} = \max\left\{\max_{i\in I}(r_i + p_i + q_i), \frac{1}{m}\sum_{i\in J_B} r_i + \frac{1}{m}\sum_{i\in \bar{J}} r_i + \frac{1}{m}\sum_{i\in J_A} q_i\right\}$$

$$\bar{J} = \underset{J\subseteq I, |J|\geq m}{Arg\max}\, LB_2^{SB}(J), \begin{cases} J_B = \{i_1,i_2,\ldots,i_m\}, i_k = Arg\min_{k,\, j\in \bar{J}} r_j \\ J_A = \{i_1,i_2,\ldots,i_m\}, i_k = Arg\min_{k,\, j\in \bar{J}} q_j \end{cases}$$

PB

$$LB^{PB} = \frac{1}{m}\sum_{j\in \bar{J}_b} r_j + \frac{1}{m}\left\{\sum_{j\in \bar{J}} p_j + \sum_{k\in \bar{K}} MH_k\right\} + \frac{1}{m}\sum_{j\in \bar{J}_a} q_j$$

$\bar{J}$ = {not marked operation nodes in the optimal max flow problem associated with $G(L^{PB})$ }

$\bar{K}$ = {interval node on which at least one unit of an operation in $\bar{J}$ is processed}

$$\begin{cases} J_B = \{i_1,i_2,\ldots,i_m\} \subseteq \bar{J} \text{ st } r_{i_1} = r_{i_2} = \ldots = r_{i_m} = \alpha \\ J_A = \{j_1,j_2,\ldots,j_m\} \subseteq \bar{J} \text{ st } q_{j_1} = q_{j_2} = \ldots = q_{j_m} = \gamma \end{cases}$$

LB^{JPPS}, LB^{ER} and LB^{PB} analytical formulations (ghost tasks)

CONCLUSION

- We have three lower bounds for the m-machines scheduling problem:
  - THE PREEMPTIVE BOUND
  - JACKSON PSEUDO PREEMPTIVE BOUND
  - THE ENERGETIC CONSTRUCTIVE BOUND
- The energetic constructive bound can be expressed similarly as JPPS and preemption

$$\alpha + \sum \text{(Truncated durations)} + \gamma$$

- We have proposed a fully quadratic algorithm for computing this bound. It can be applied directly to the CUSP
- We improve the complexities of Checker and adjustment algorithms proposed by Baptiste et al.
- We characterize mathematically the three bounds. They are very similar.
- In practice the three bounds are generally equal.

OPEN QUESTIONS:

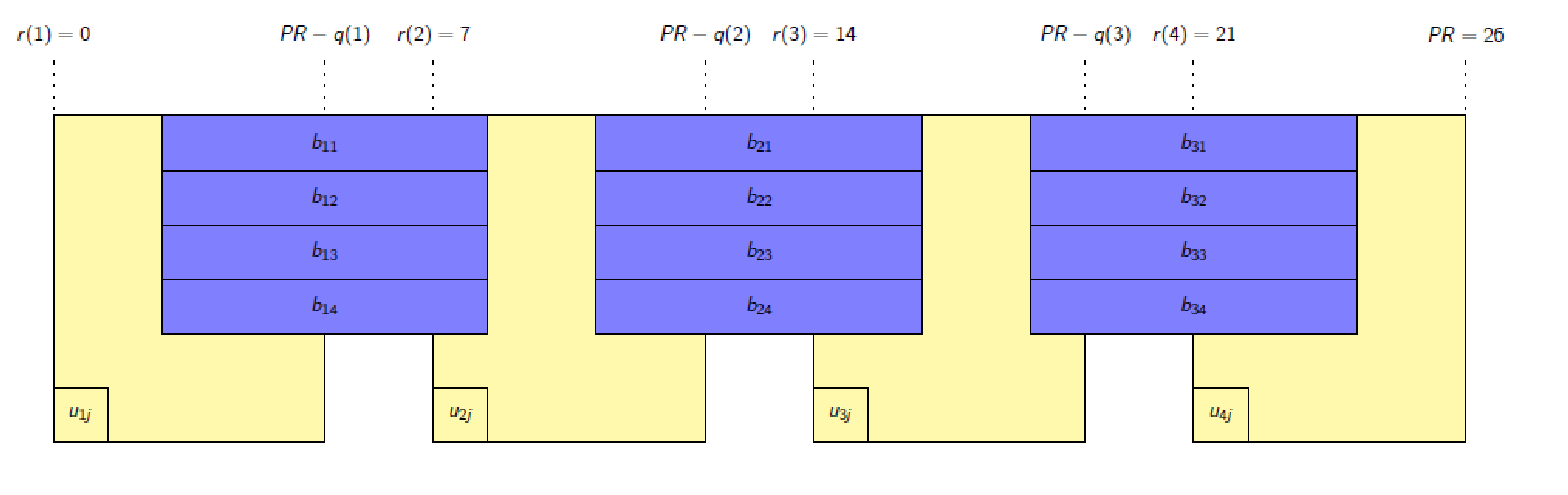
- Can we get rid of Ackermann coefficient (generally equal to 3) in practice? In theory?
- Can we improve the data structure based on Range trees?

WORKS IN PROGRESS

- EXTENSION OF ENERGY NOTION (See our talk ROADEF 2022) TO THE CUSP, THEN TO RCPSP BY USING JPS, JPPS AND LLB.

- THEORETICAL GAP BETWEEN THE THREE BOUNDS (collaboration with Claire Hanen, gap : pmax) Carlier, Hanen, PMS 2022.

- **Illustration: A bandaneon data**



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Thank you for your attention ?