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On the state of the art in proactive/reactive project scheduling

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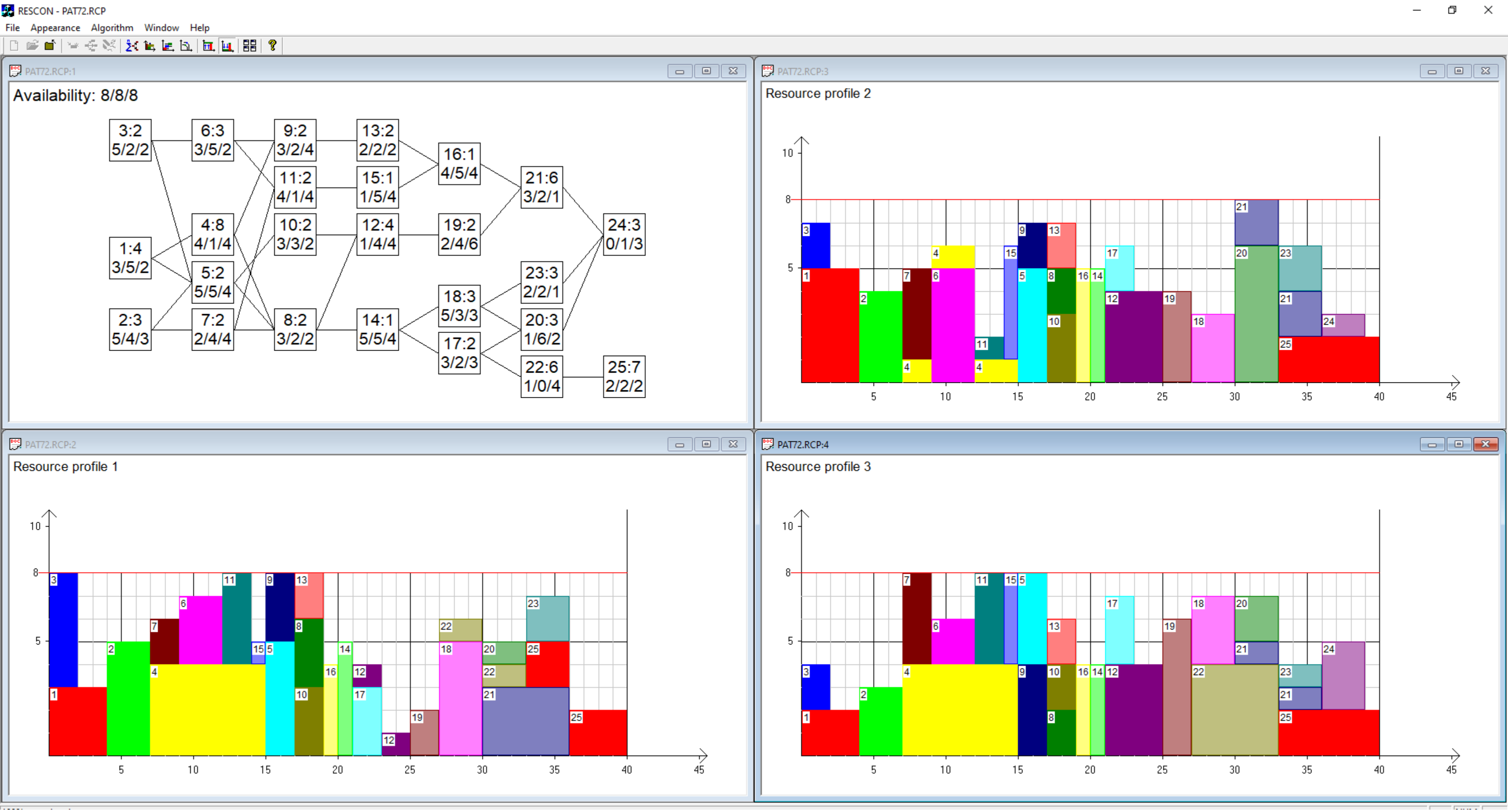
https://feb.kuleuven.be/rescon/



Idea of robust project scheduling

- Generate a baseline schedule that incorporates a degree of anticipation of variability during project execution and/or information about the reactive scheduling approach to be used
- Objectives:
  - Solution robustness (stability):
    - Measure of the difference between the baseline schedule and the realized schedule
  - Quality robustness:
    - Sensitivity of the schedule performance in terms of the objective value (makespan) other than stability

The stochastic RCPSP

- Classes of scheduling policies:
  - Resource-based policies
  - Early-start policies
  - Preselective policies
  - Linear preselective policies
  - Activity-based policies
  - Pre-processing policies
- Drawback: no baseline schedule, thus no measure of solution robustness

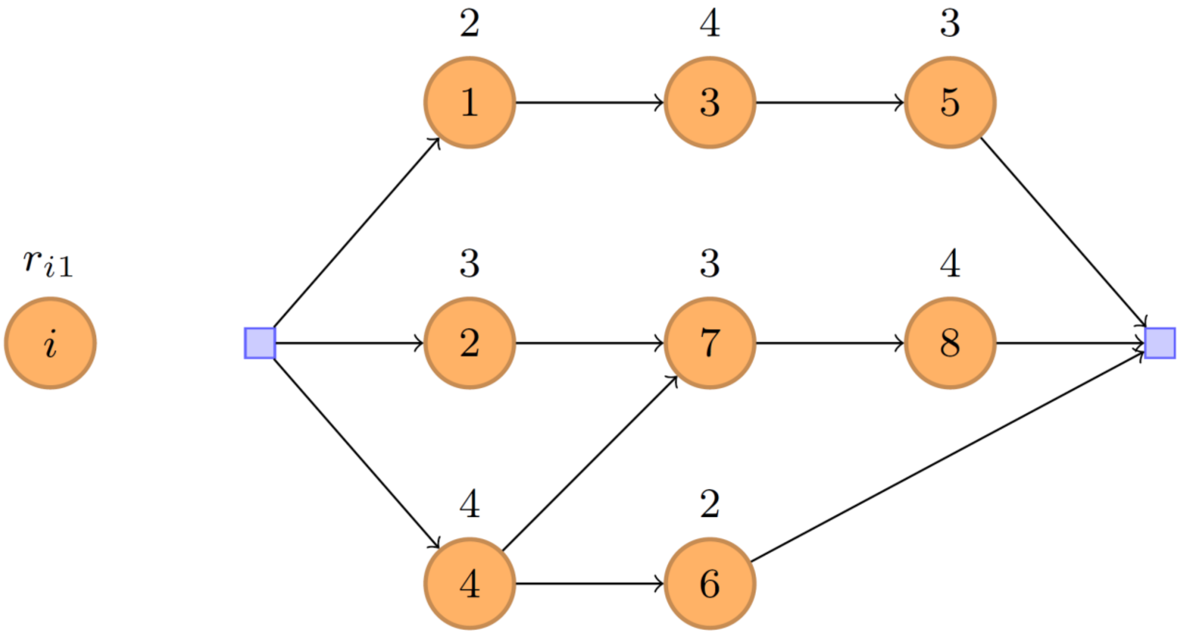
Proactive-reactive project scheduling

- Proactive scheduling:
  - Construct a robust baseline schedule that accounts for the available statistical knowledge of uncertainty and that is protected as best as possible against disruptions
- Reactive scheduling:
  - Revise or reoptimize a schedule whenever a schedule breakage occurs

The proactive and reactive RCPSP

An instance of the RCPSP

This instance contains eight activities and two dummy activities.
This instance has one resource type of availability 8.



Example for $\tilde{\mathrm{p}}$

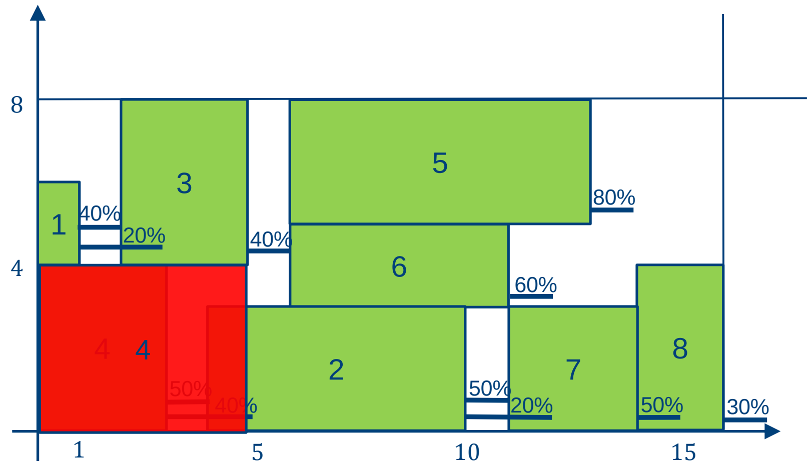
	$\pi(\tilde{p}_i = p)$									$w_{i,0}$
	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	
$\tilde{p}_0$	1	-	-	-	-	-	-	-	-	-
$\tilde{p}_1$	-	0.4	**0.4**	0.2	-	-	-	-	-	4
$\tilde{p}_2$	-	-	-	-	-	-	0.3	0.5	**0.2**	4
$\tilde{p}_3$	-	-	-	**0.6**	0.4	-	-	-	-	7
$\tilde{p}_4$	-	-	-	0.1	0.5	**0.4**	-	-	-	1
$\tilde{p}_5$	-	-	-	-	-	-	-	**0.2**	0.8	4
$\tilde{p}_6$	-	-	-	-	-	**0.4**	0.6	-	-	1
$\tilde{p}_7$	-	-	-	0.5	**0.5**	-	-	-	-	1
$\tilde{p}_8$	-	-	**0.7**	0.3	-	-	-	-	-	1
$\tilde{p}_9$	1	-	-	-	-	-	-	-	-	38

Total number of combinations:
$2^5 \times 3^3 = 864$

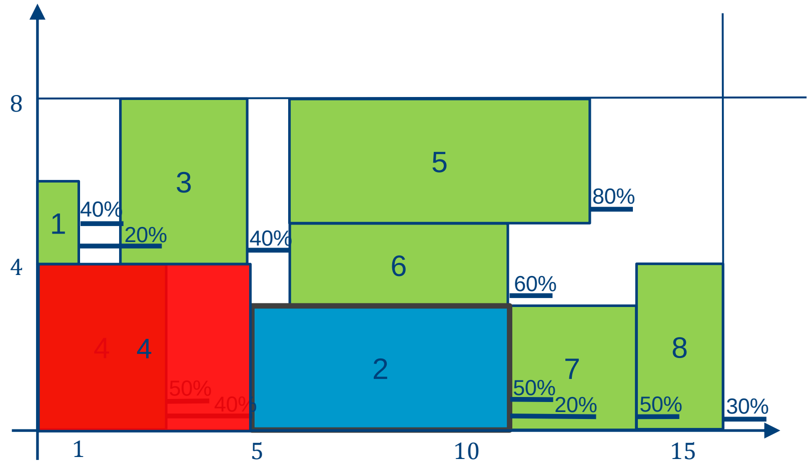
An example realization

$\mathbf{p_1} = (0,2,8,3,5,7,5,4,2,0)$
$\pi(\tilde{\mathbf{p}} = \mathbf{p_1}) = 0.054\%$

Proactive and reactive scheduling



Proactive solution



Reactive scheduling

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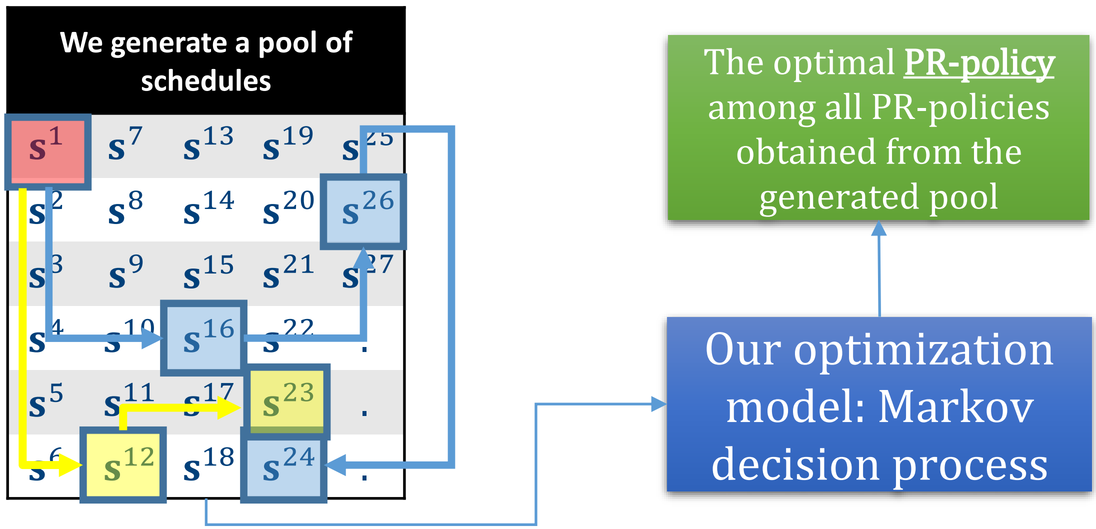
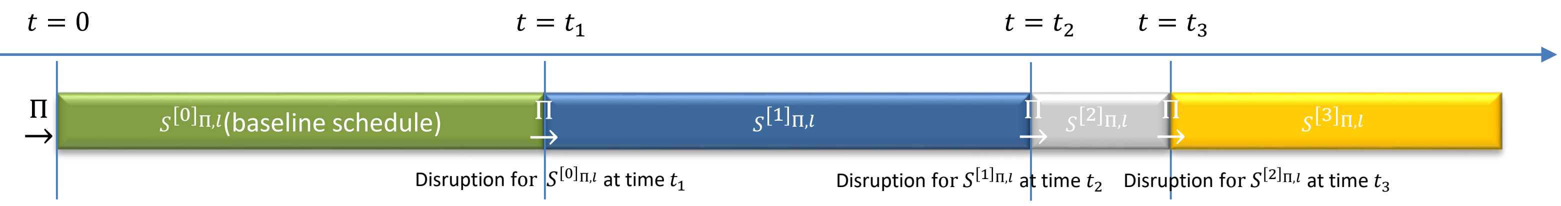
The basic idea



Objective: to minimize the expected value of
(cost of the baseline schedule
+ cost of a series of reactions)

A PR-policy

PR-policy Π for realization $\mathbf{p}_l$



A chain of reactions dictated by PR-policy Π for $\mathbf{p}_l$

$$CH_{\Pi,l}: S^{[0]}{}_{\Pi,l} \xrightarrow{t_1} S^{[1]}{}_{\Pi,l} \xrightarrow{t_2} \dots \xrightarrow{t_{\nu_{\Pi,l}}} S^{[\nu_{\Pi,l}]}{}_{\Pi,l}$$

The number of reactions for the combination (Π, l)

$$\Pi = \{CH_{\Pi,1}, \dots, CH_{\Pi,|\mathbf{p}|}\}$$

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An example

PR-policy $\mathbf{\Pi_1}$

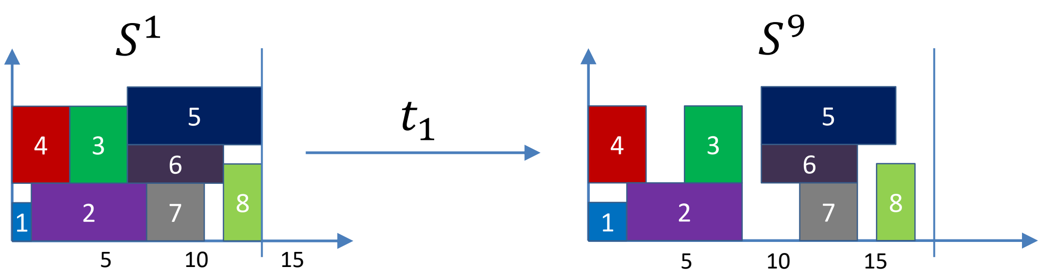
$\mathbf{p_1} = (0,2,8,3,5,7,5,4,2,0)$

$CH_{\Pi_1,1}: S^1 \xrightarrow{t_1=1} S^9$

…

$\mathbf{\Pi_1} = \{CH_{\Pi_1,1}, \dots, CH_{\Pi_1,|\mathbf{p}|}\}$

S^1	S^2	S^3	S^4	S^5	S^6	S^7	S^8	S^9	S^{10}
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
1	1	0	1	5	0	7	4	2	7
3	3	4	4	3	3	3	3	5	5
0	0	4	0	0	7	0	0	0	9
6	6	7	7	7	7	7	7	9	14
6	6	7	7	7	12	5	7	9	14
7	8	7	8	12	12	14	12	11	15
11	13	13	12	15	15	17	15	15	20
13	15	15	15	17	18	19	18	18	23



Deadchains

A chain of reactions dictated by PR-policy Π for $\mathbf{p}_l$

$CH_{\Pi,l}: S^{[0]_{\Pi,l}} \xrightarrow{t_1} S^{[1]_{\Pi,l}} \xrightarrow{t_2} \ldots \xrightarrow{t_{\nu_{\Pi,l}}} S^{[\nu_{\Pi,l}]_{\Pi,l}}$

What if this schedule is not feasible for $\mathbf{p}_l$?

What is a deadchain?

$S^{[\nu_{\Pi,l}]_{\Pi,l}}$ is a deadend Ω if it is not feasible for realization $\mathbf{p}_l$.
$CH_{\Pi,l}$ is a deadchain if it contains a deadend.

$\gamma_{\Pi,l} \in \{0,1\}$

$\gamma_{\Pi,l} = 1$ if chain $CH_{\Pi,l}$ is a deadchain
$\gamma_{\Pi,l} = 0$ otherwise

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Cost of a chain (a general function)

$$f(\Pi, l) = g\left(s^{[0]_{\Pi,l}}\right) + \sum_{k=1}^{\nu_{\Pi,l}} e\left(S^{[k-1]_{\Pi,l}}, S^{[k]_{\Pi,l}}, k\right) + h(\gamma_{\Pi,l})$$

Cost of the baseline schedule

Cost of reactions

Cost of deadchains

Conceptual formulation

$$\text{P}: \min_{\Pi \in \Xi} \sum_{l=1}^{|\mathbf{p}|} \pi(\tilde{\mathbf{p}} = \mathbf{p}_l) f(\Pi, l)$$

Cost of a chain

Probability of occurrence

Cost of a chain (an example: used in our method)

$$w_b S_{n+1}^{[0]_{\Pi,l}} + \sum_{k=1}^{\nu_{\Pi,l}} \left(\sum_{i \in N} w_{ik} \left| S_i^{[k]_{\Pi,l}} - S_i^{[k-1]_{\Pi,l}} \right| + w_r \right) + w_d \gamma_{\Pi,l}$$

The cost per unit time of the completion of the baseline schedule

The weight of each activity

The fixed cost of a reaction

The cost of a deadchain

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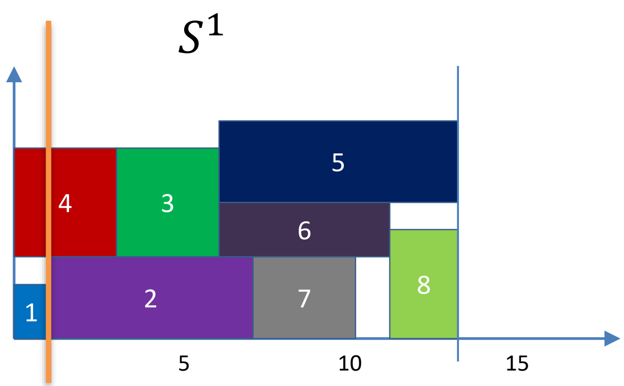
Representation of states in Model 1

(S, t, O, v)

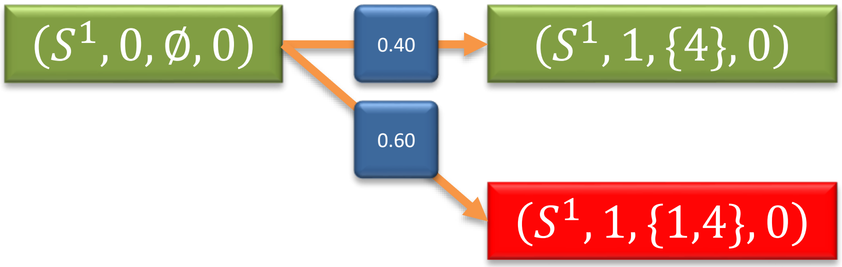
- S: The current schedule
- t: The current decision moment
- O: The set of ongoing activities
- v: The number of reactions that occurred so far

An example (S^1)



$\rightarrow (S^1, 1, \{4\}, 0)$

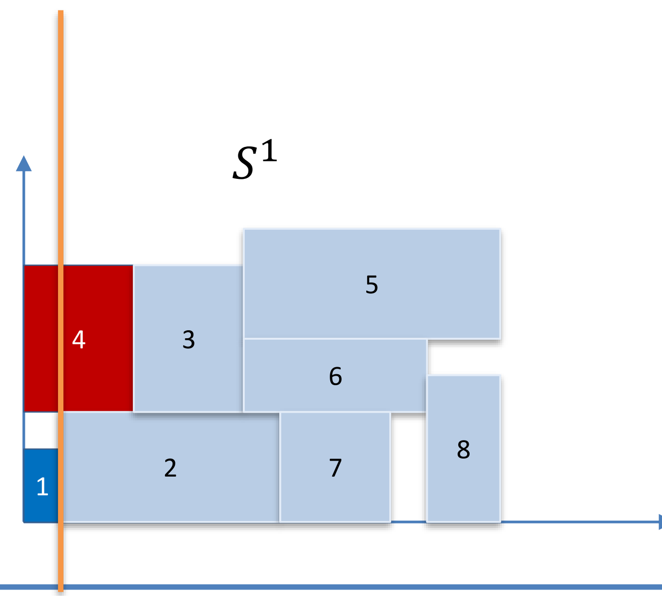
A chance transition



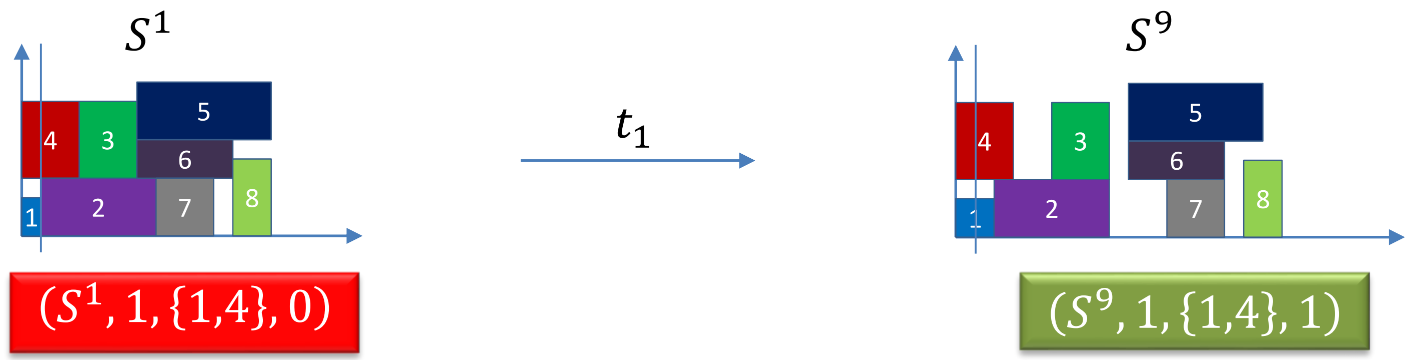
Continue with the same schedule

Infeasible: needs reaction

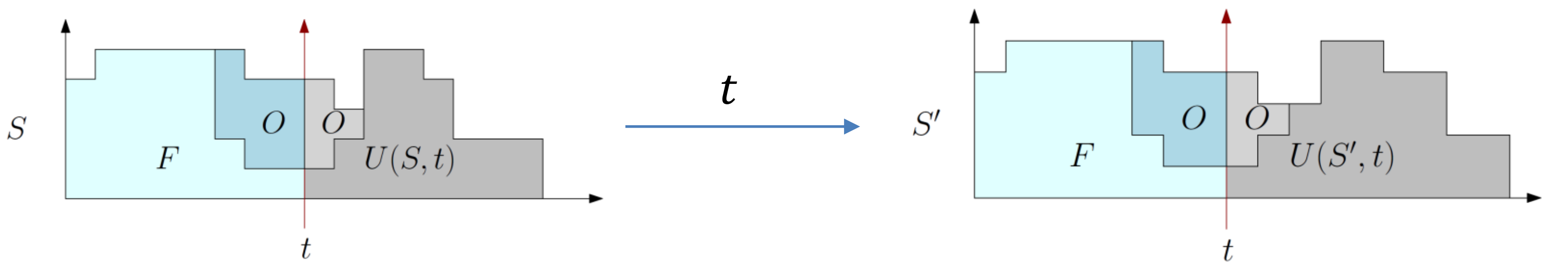
	$\pi(\tilde{p}_i = p)$								
	0	1	2	3	4	5	6	7	8
$\tilde{p}_0$	**1**	-	-	-	-	-	-	-	-
$\tilde{p}_1$	-	0.4	**0.4**	0.2	-	-	-	-	-
$\tilde{p}_2$	-	-	-	-	-	-	0.3	0.5	**0.2**
$\tilde{p}_3$	-	-	-	**0.6**	0.4	-	-	-	-
$\tilde{p}_4$	-	-	-	0.1	0.5	**0.4**	-	-	-
$\tilde{p}_5$	-	-	-	-	-	-	-	**0.2**	0.8
$\tilde{p}_6$	-	-	-	-	-	**0.4**	0.6	-	-
$\tilde{p}_7$	-	-	-	0.5	**0.5**	-	-	-	-
$\tilde{p}_8$	-	-	**0.7**	0.3	-	-	-	-	-
$\tilde{p}_9$	**1**	-	-	-	-	-	-	-	-



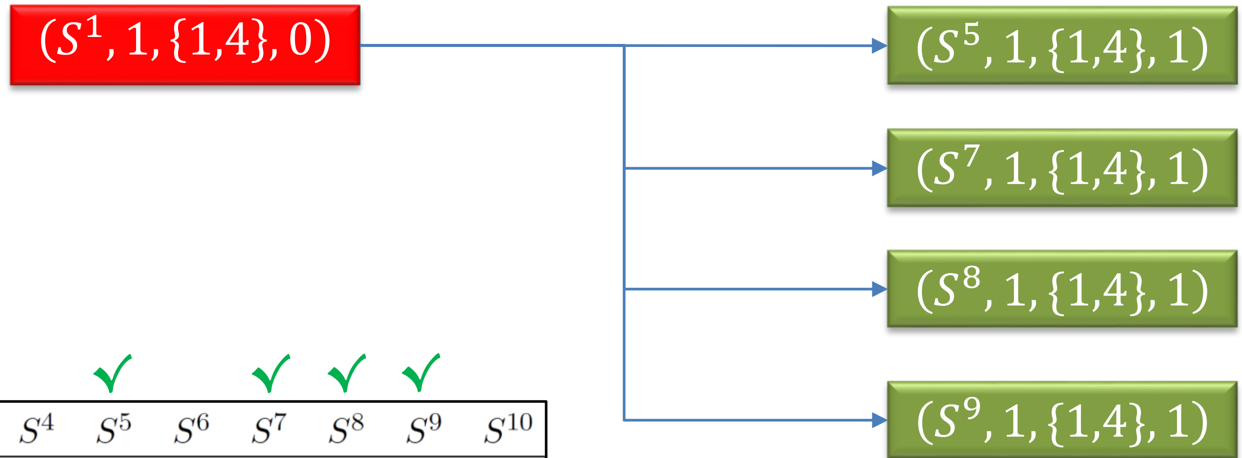
A valid reaction



A valid reaction (general rules)

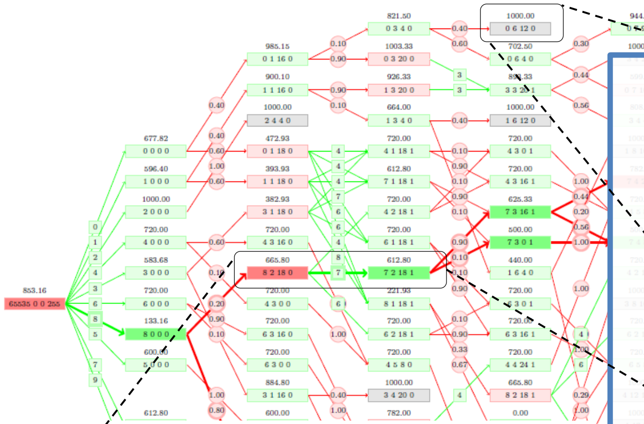


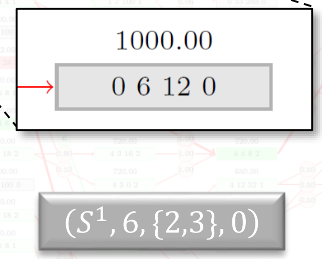
Reaction possibilities for $(S^1, 1, \{1,4\}, 0)$



S^1	S^2	S^3	S^4	✓ S^5	S^6	✓ S^7	✓ S^8	✓ S^9	S^{10}
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
1	1	0	1	5	0	7	4	2	7
3	3	4	4	3	3	3	3	5	5
0	0	4	0	0	7	0	0	0	9
6	6	7	7	7	7	7	7	9	14
6	6	7	7	7	12	5	7	9	14
7	8	7	8	12	12	14	12	11	15
11	13	13	12	15	15	17	15	15	20
13	15	15	15	17	18	19	18	18	23

How to read a state?

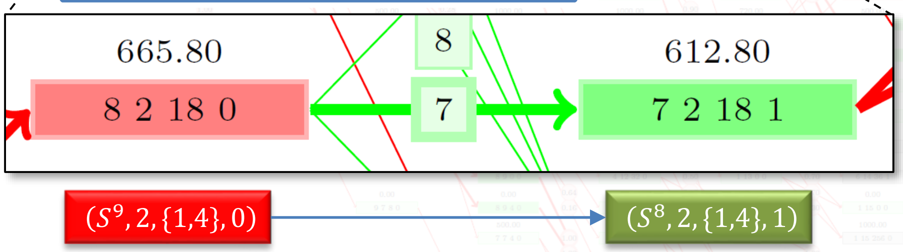


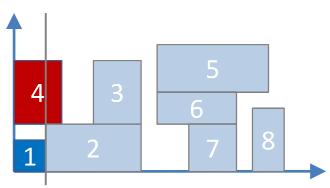


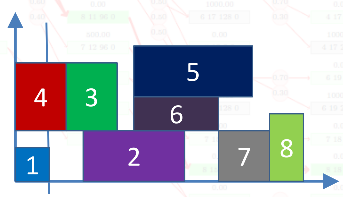
1000: cost of the state

0: schedule S^1 $(1 \rightarrow S^2, 2 \rightarrow S^3, \ldots)$
6: time 6
12: {2,3} $(2^2 + 2^3 = 12)$
0: no reaction

An optimal reaction in the optimal policy

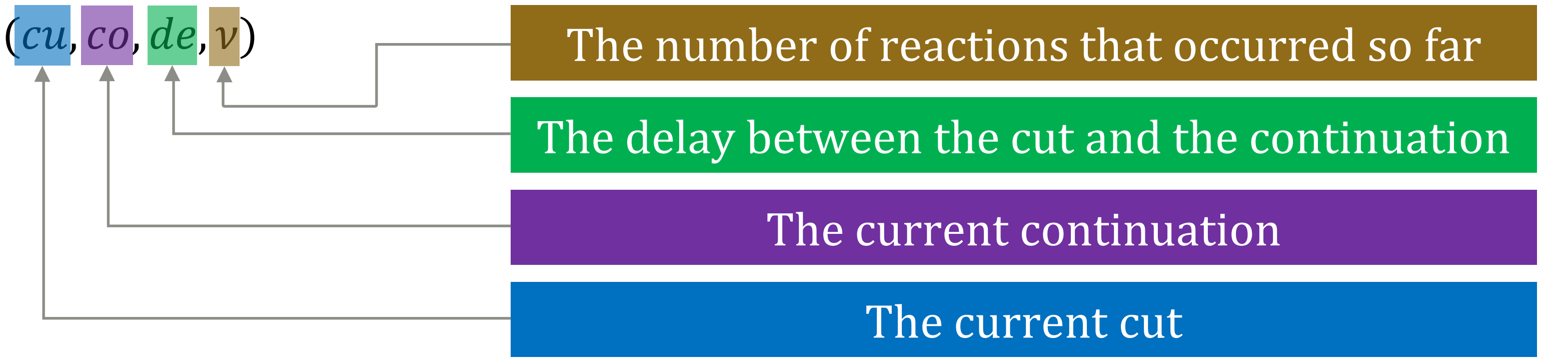






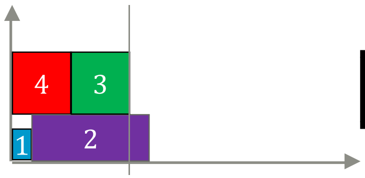
The cost of this reaction:
$$|4-2|w_2 + |3-5|w_3 + |7-9|w_5 + |7-9|w_6 + |12-11|w_7 + w_r =$$
$$8 + 14 + 8 + 2 + 1 + 20$$
= <u>53</u>

Representation of states in Model 3



A cut

(F, O, el^{O})

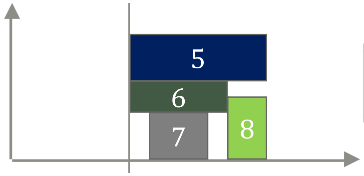


$cu = (\{0,1,3,4\}, \{2\}, (*,*,5,*,*,*,*,*,*,*))$

- The vector of the elapsed times of ongoing activities
- The set of ongoing activities
- The set of finished activities

A continuation

(I, rs^{I})



$co = (\{5,6,7,8,9\}, (*,*,*,*,*,0,0,1,5,7))$

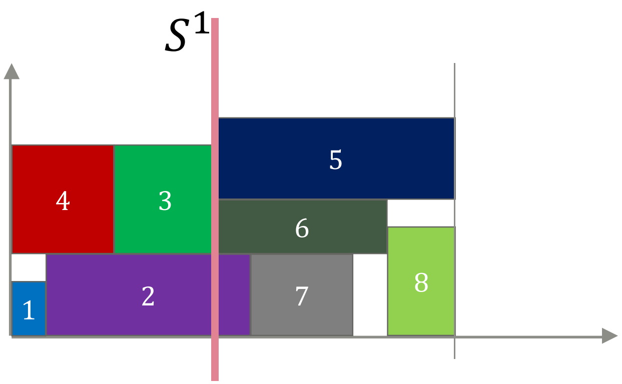
- The vector of the relative starting times of idle activities
- The set of idle activities

Representation of states in Model 3

(cu, co, de, v)

- v: The number of reactions that occurred so far
- de: The delay between the cut and the continuation
- co: The current continuation
- cu: The current cut

An example $(S^1, 6, \{2\}, 0)$



$cu_1 = (\{0,1,3,4\}, \{2\}, (*,*,5,*,*,*,*,*,*,*))$

$co_1 = (\{5,6,7,8,9\}, (*,*,*,*,*,0,0,1,5,7))$

$(S^1, 6, \{2\}, 0) \rightarrow (cu_1, co_1, 0,0)$

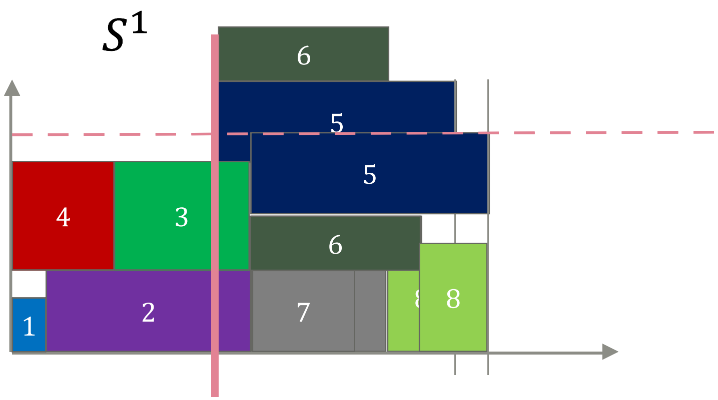
State space

- We first generate all possible cuts that can be obtained from **S**
- We generate all possible continuations that can be obtained from **S**
- $de \in \{0,1\}$

- There is a state for each combination (cu, co, de, v) such that

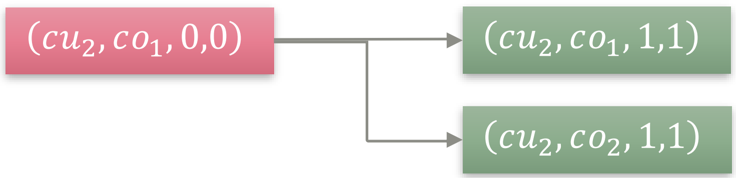
$$(F \cup O) \cap I = \emptyset \text{ and } (F \cup O) \cup I = N$$

An example $(S^1, 6, \{2,3\}, 0)$



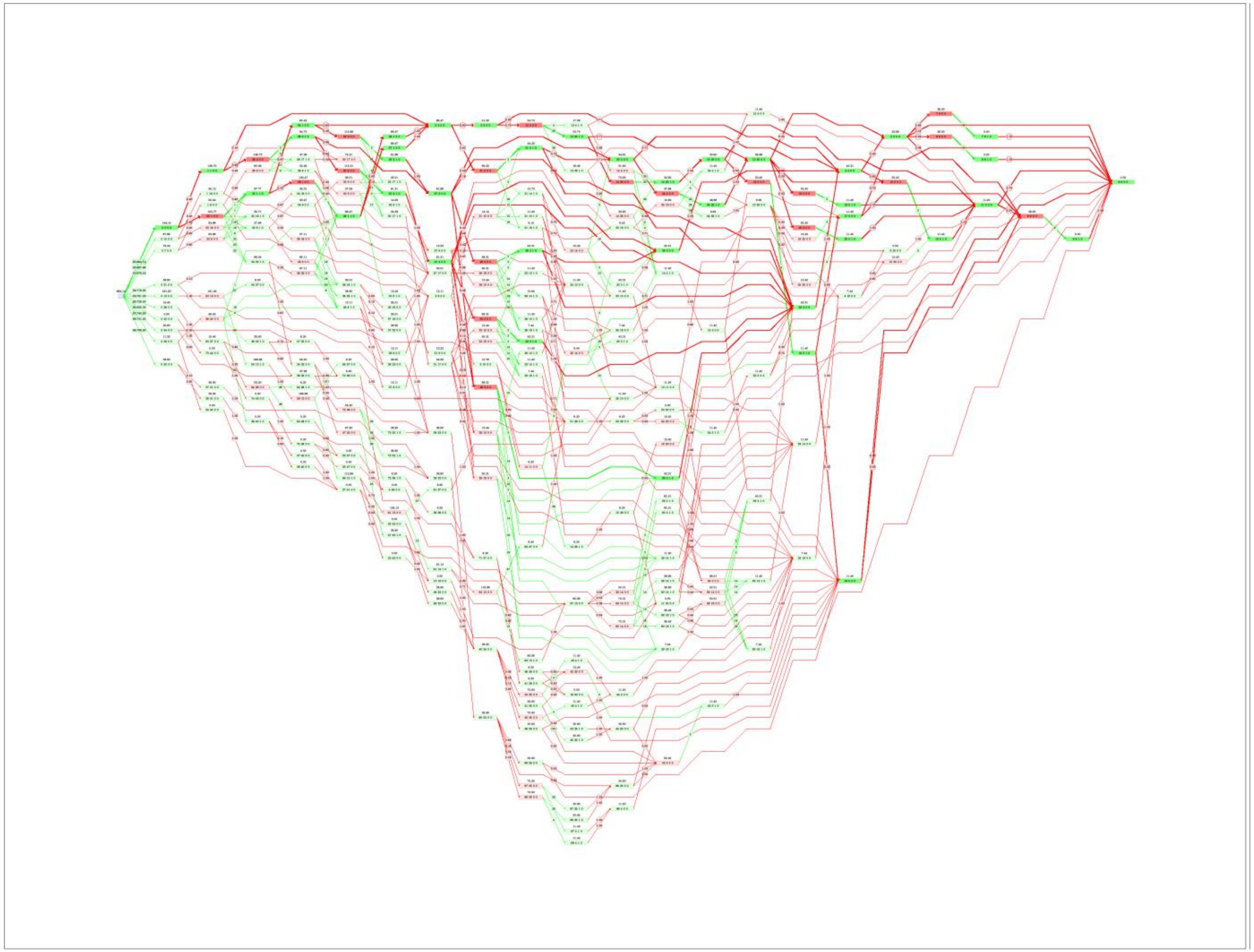
$cu_2 = (\{0,1,4\}, \{2,3\}, (*,*,5,3,*,*,*,*,*,*))$

$co_1 = (\{5,6,7,8,9\}, (*,*,*,*,*,0,0,1,5,7))$



$co_2 = (\{5,6,7,8,9\}, (*,*,*,*,*,0,0,0,5,7))$

Model 3: Markov decision process



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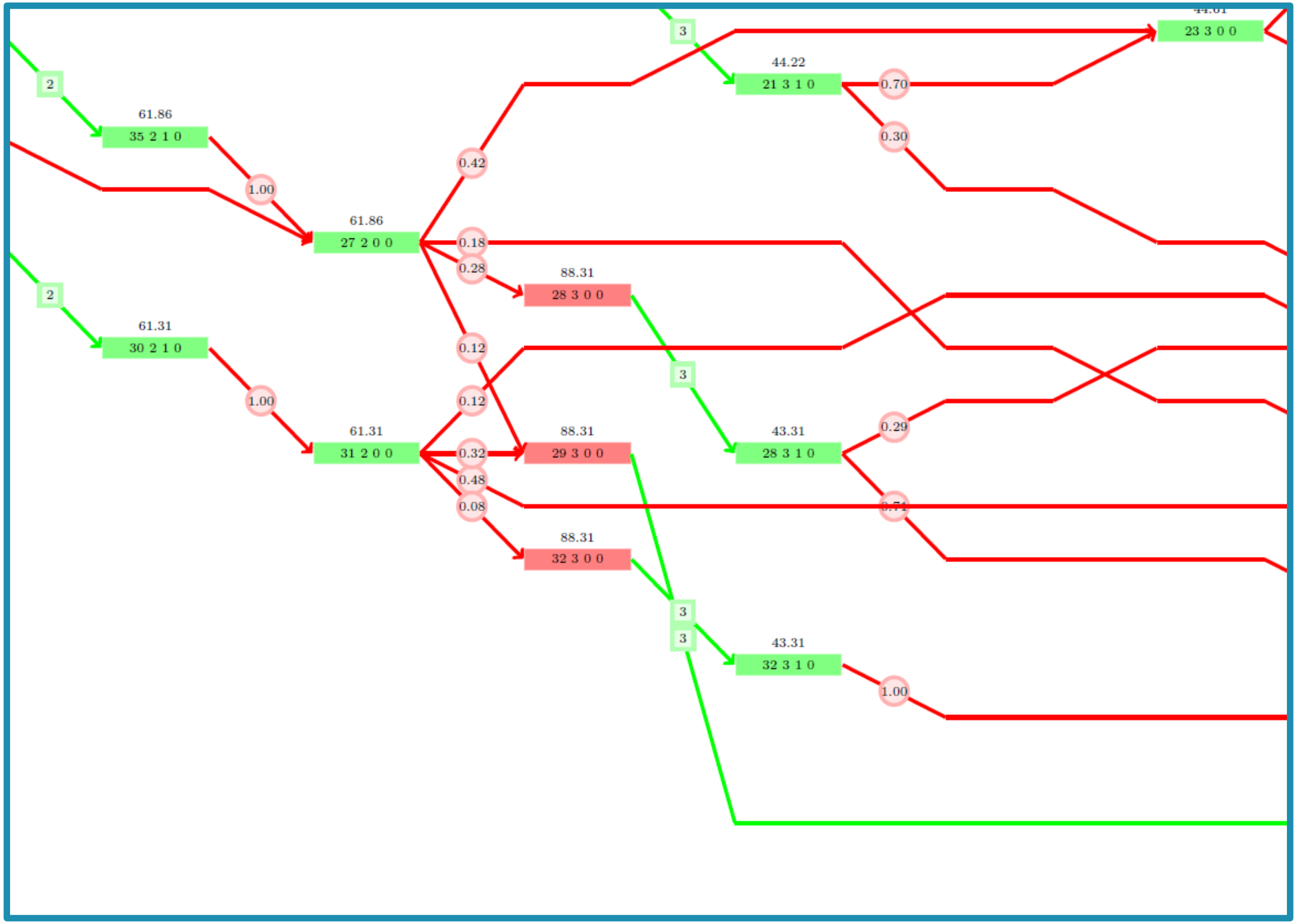
Model 3: optimal PR-policy



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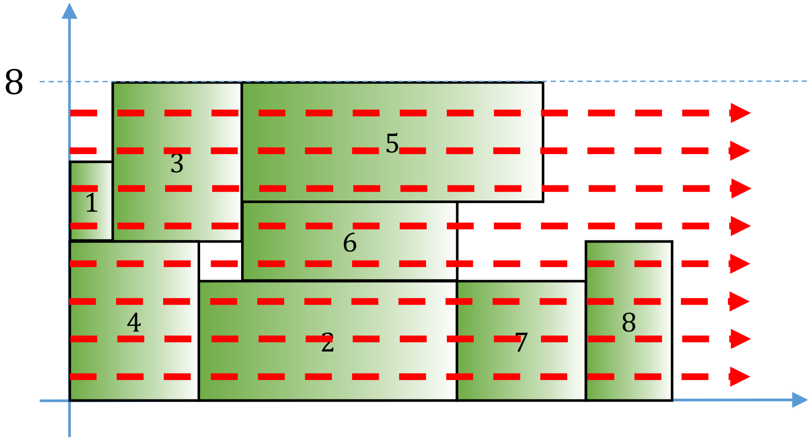
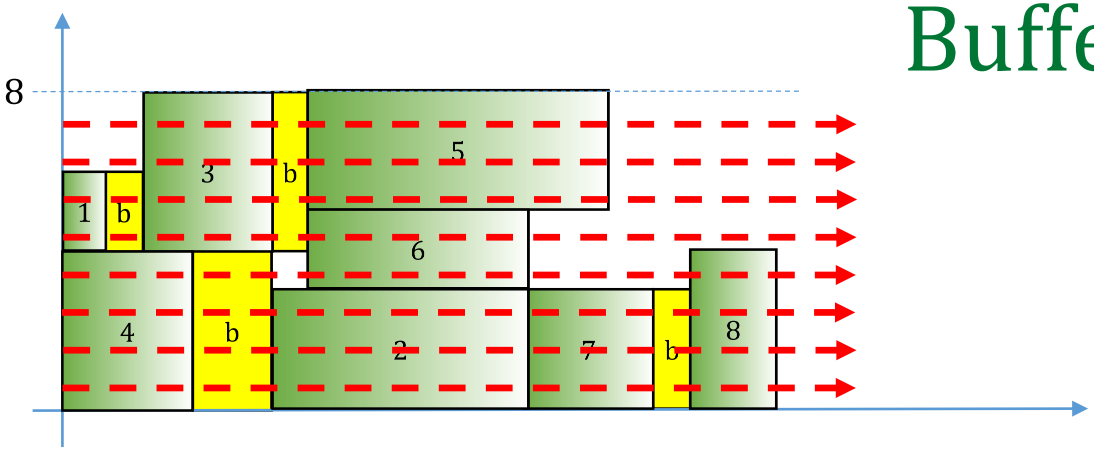
Selection- and buffer-based reactions





Selection-based ✓

Buffer-based ✓

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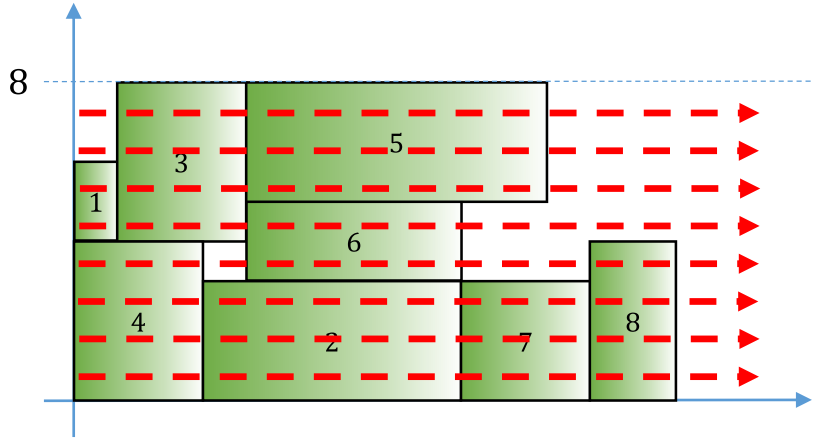
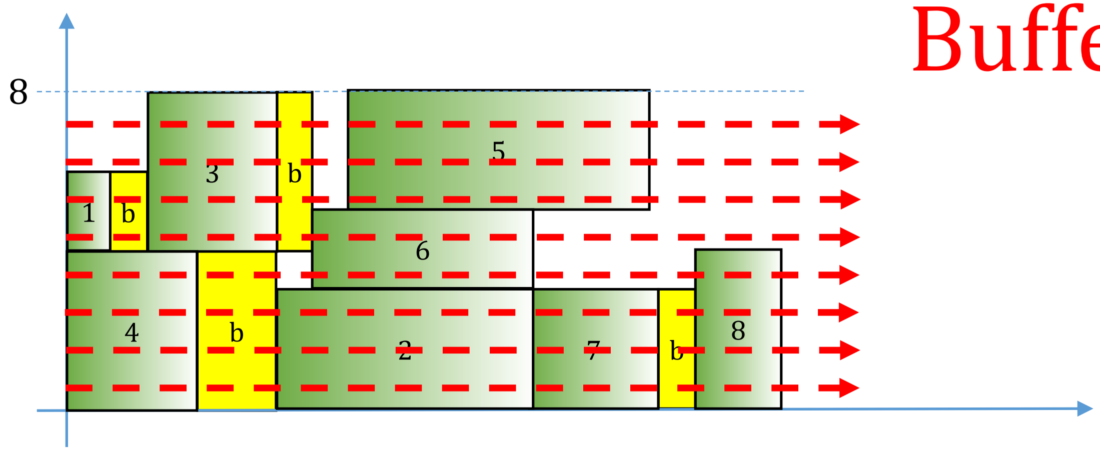
A selection- but not buffer-based reaction





Selection-based ✓

Buffer-based ✗

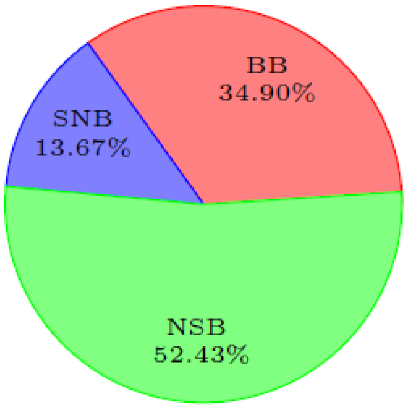
Computational results

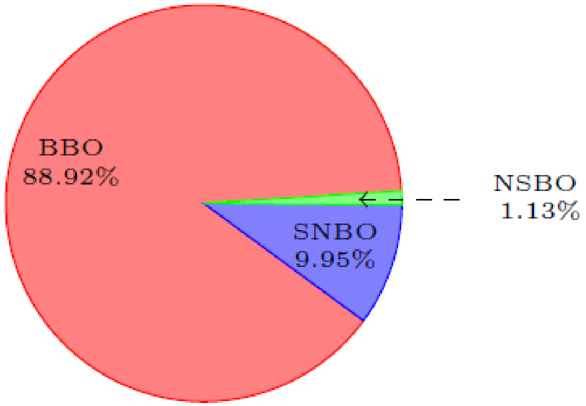
Three exclusive classes of reactions

The class of non-selection-based (NSB) reactions
The class of selection but not buffer-based (SNB) reactions
The class of buffer-based (BB) reactions

Selection-based

$w_b = 25$ and $w_r = 0$





In the optimal PR-policy

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References

- Davari, M., 2017, Contributions to complex project and machine scheduling problems, PhD dissertation Faculty of Economics and Business, KU Leuven, number 554.
- Davari, M. & E. Demeulemeester, 2019, The proactive and reactive resource-constrained project scheduling problem, Journal of Scheduling, 22 (2), 211-237.
- Davari, M. & E. Demeulemeester, 2019, Important classes of reactions for the proactive and reactive resource-constrained project scheduling problem, Annals of Operations Research, 274 (1-2), 187-210.

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