Combinatorial Benders Approach for the Quay Crane Scheduling Problem

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Outline

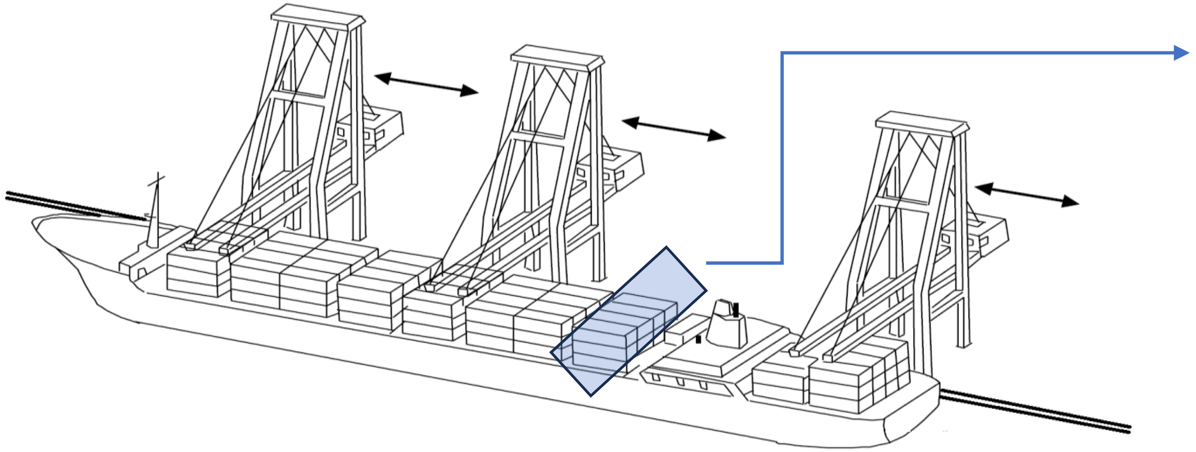
1. Problem Description
2. Mathematical formulation
3. Combinatorial Benders Decomposition
4. Computational Results
5. Conclusion

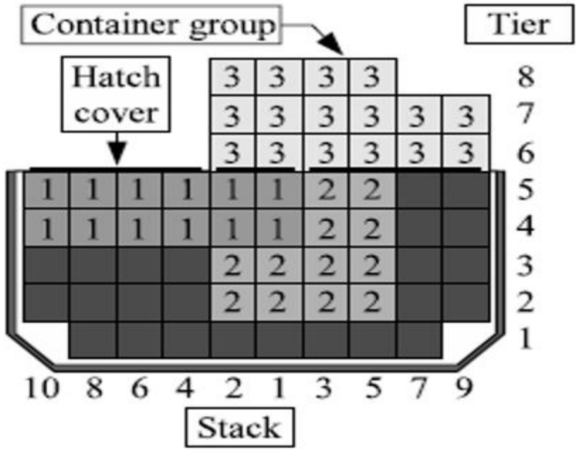
1. Problem Description

- Definition: Schedule quay cranes (abbr. **QC**s) to unload or load containers from or into the vessel with respect to a given objective function(e.g. minimizing the maximum completion time).
- According to the classification of Bierwirth and Meisel (2010, 2015), class [group,prec| ready,pos,move|cross,save|max(compl)]

Reference :
Bierwirth and Meisel (2009)

QCs can move
multi-directionally





Difficulties : 1) spatial constraints: non-crossing; 2) task precedence

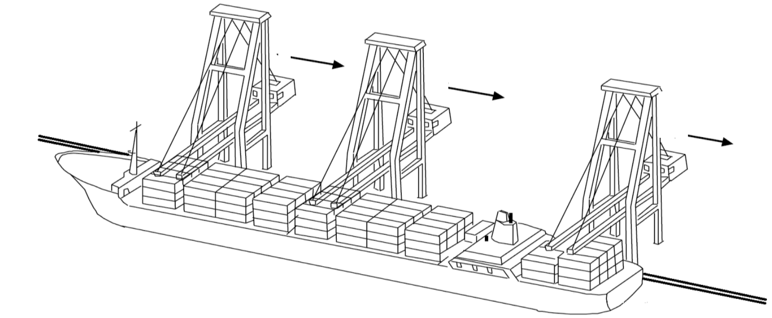
Result in complex operation sequence constraints.
Even small-scale instances hard to be solved to optimal – 2 QCs, 10 bays – 2 hours

1. Problem Description

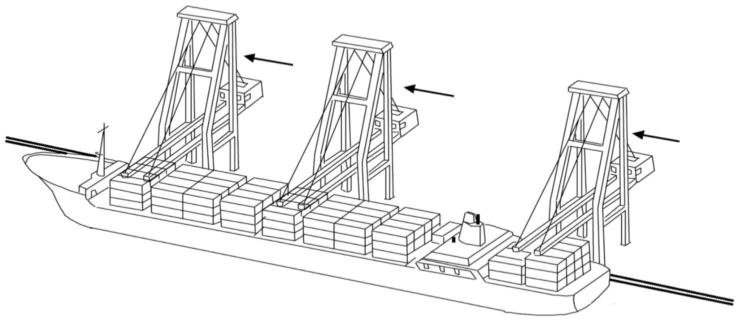
❖ Commonly adopted approximation strategy

➢ **Unidirectional** schedule

Forces QCs to move unidirectionally only.



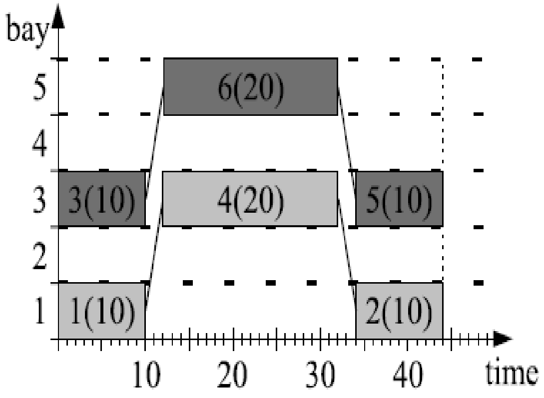
or

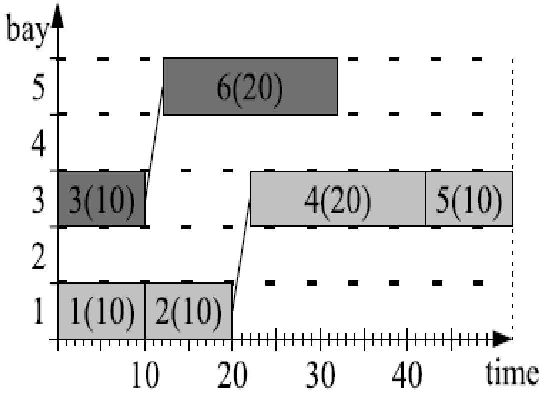


Reference : Bierwirth and Meisel (2009), Chen et al. (2014) , Chen et al. (2017) , Sun et al. (2019)

➢ **Bidirectional** schedule

Allow QCs to have two-stage movements at two directions, i.e. left-to-right and then right-to-left





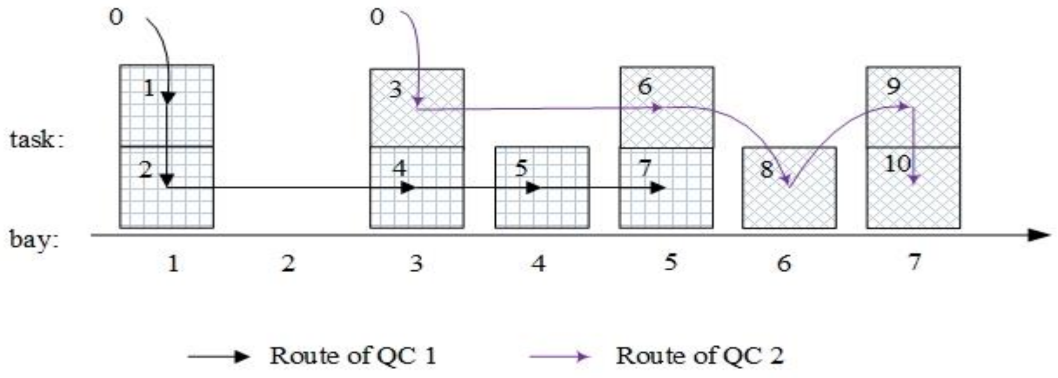
Bidirectional **vs** **Unidirectional**

Reference : Sun, D., Tang, L., Baldacci, R., Chen, Z. A Decomposition Method for the Group-Based Quay Crane Scheduling Problem. *INFORMS Journal on Computing*, 2024, 36(2), 543–570.

1. Problem Description

❖ Commonly adopted approximation strategy

Unidirectioanl & bidirectional QCSP



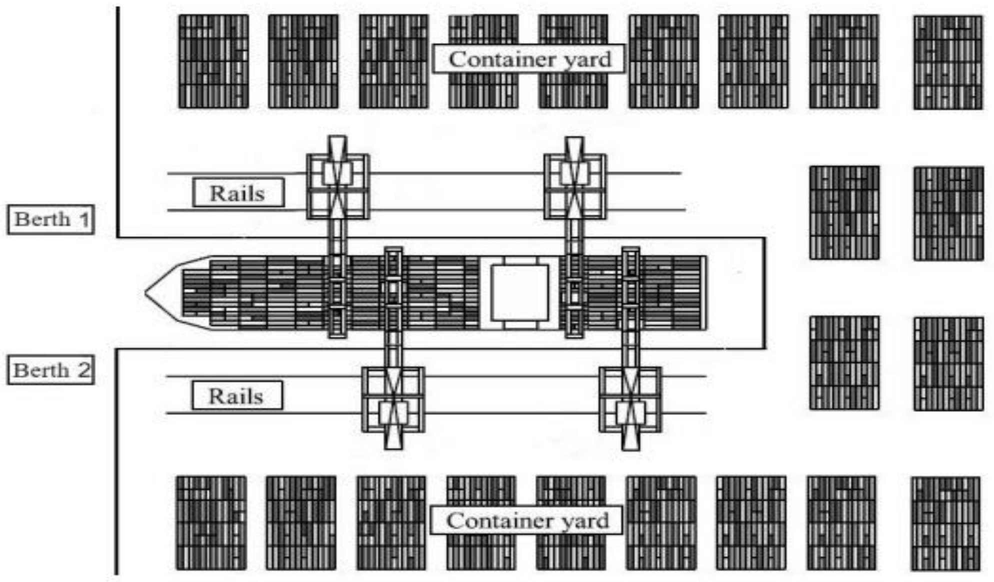
➢Advantages:

1) Assignment determines sequence

2) Concise precedence constraints

3) Fast solving and near-optimal solution

QCSP at indented berth



➢ Disadvantages:

1) Lose optimality

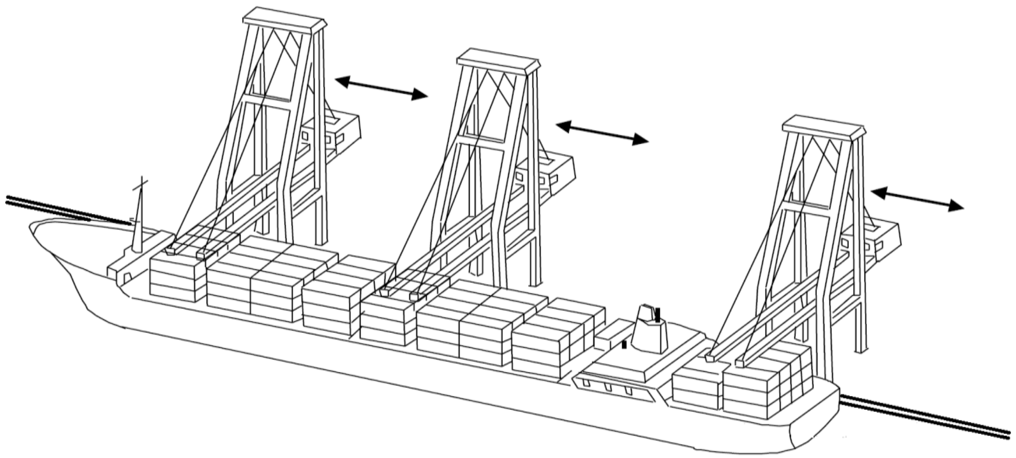
2) Hard to be generalized

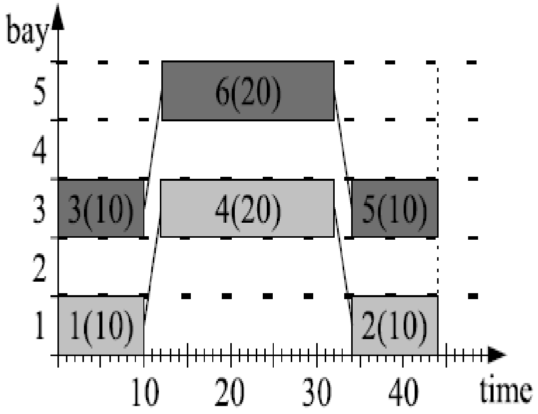
1. Problem Description

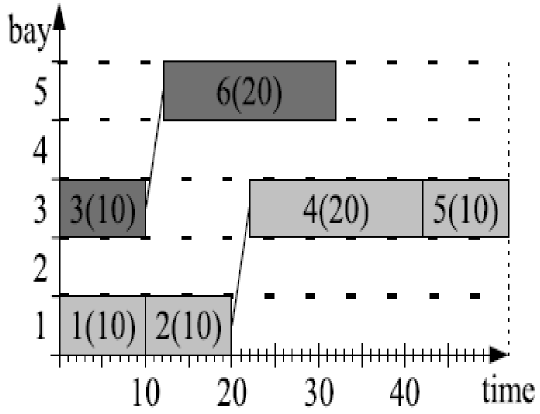
❖ Our contribution

- A new compact formulation for the QCSP which is fit for most QCSP variants.
- Combinatorial Benders approach capable to optimally solve medium-sized benchmark instances.
- Various valid inequalities and several types of combinatorial cuts to accelerate the convergence.

Multi-directional – hard to solve







2. Mathematical formulation

❖ Task assignment and sequencing

- y_{ik}: binary variable equal to 1 if task $i \in \Omega$ is processed by QC $k \in K$, 0 otherwise;
- x_{ij}^k: binary variable equal to 1 if task $j \in \Omega$ is processed after $i \in \Omega$, $i \neq j$, by QC $k \in K$, 0 otherwise;

❖ Time allocation (considering QC collisions)

- W: nonnegative continuous variable representing the makespan;
- D_i: nonnegative continuous variable representing the completion time of task $i \in \Omega$;
- C_k: nonnegative continuous variable representing the completion time of QC $k \in K$;
- z_{ij}: binary variable equal to 1 if $i \in \Omega$ is completed before $j \in \Omega$, $i \neq j$, starts, 0 otherwise;

2. Mathematical formulation

Task assignment and sequencing

$$(F) \quad \min \quad W \tag{1a}$$

$$s.t. \sum_{j \in \Omega_T} x^k_{0,j} = 1, \quad \forall k \in K \tag{1b}$$

$$\sum_{i \in \Omega_0} x^k_{i,T} = 1, \quad \forall k \in K \tag{1c}$$

$$\sum_{j \in \Omega_T} x^k_{ij} = y_{ik} \quad \forall i \in \Omega, k \in K \tag{1d}$$

$$\sum_{i \in \Omega_0} x^k_{ij} = y_{jk}, \quad \forall j \in \Omega, k \in K \tag{1e}$$

$$\sum_{k \in K} y_{ik} = 1, \quad \forall i \in \Omega, \tag{1f}$$

Δ^{uv}_{ij} the minimum time to elapse between the processing of tasks *i* and *j* if assigned to QCs *u* and *v*, respectively

Time allocation

$$D_i + t_{ij} + p_{jk} - D_j \le M\left(1 - \sum_{k \in K} x^k_{ij}\right), \quad \forall i, j \in \Omega \tag{1g}$$

$$D_i + p_{jk} - D_j \le M(1 - z_{ij}), \quad \forall i, j \in \Omega, l_i \ne l_j \tag{1h}$$

$$D_j - p_{jk} - D_i \le M z_{ij}, \quad \forall i, j \in \Omega, l_i \ne l_j \tag{1i}$$

$$y_{iu} + y_{jv} \le 1 + z_{ij} + z_{ji}, \quad \forall (i, j, v, w) \in \Theta \tag{1j}$$

$$D_i + \Delta^{uv}_{ij} + p_{jk} - D_j \le M(3 - z_{ij} - y_{iu} - y_{jv}), \quad (i, j, v, w) \in \Theta \tag{1k}$$

$$D_j + \Delta^{uv}_{ij} + p_{ik} - D_i \le M(3 - z_{ji} - y_{iu} - y_{jv}), \quad (i, j, v, w) \in \Theta \tag{1l}$$

$$z_{ij} + z_{ji} = 1, \quad \forall (i, j) \in \Psi \setminus \Phi \tag{1m}$$

$$z_{ij} = 1, z_{ji} = 0, \quad \forall (i, j) \in \Phi \tag{1n}$$

$$r_k - D_j + t^k_{0j} + p_{jk} \le M(1 - x^k_{0j}), \quad \forall j \in \Omega, k \in K \tag{1o}$$

$$D_j + t^k_{jT} - C_k \le M(1 - x^k_{jT}), \quad \forall j \in \Omega, k \in K \tag{1p}$$

$$C_k \le d_k, \quad \forall k \in K \tag{1q}$$

$$C_k \le W, \quad \forall k \in K \tag{1r}$$

$$\sum_{i \in \Omega^F_k} y_{ik} = 0, \quad \forall k \in Q \tag{1s}$$

$$x^k_{ij}, y_{ik}, z_{ij} \in \{0, 1\}, D_i, C_k, W \ge 0, \quad \forall i, j \in \Omega, k \in K. \tag{1t}$$

3. Combinatorial Benders approach

Task assignment and sequencing

$$
\begin{aligned}
(F)\quad & \min \quad W && \text{(1a)}\\
s.t.\ & \sum_{j\in\Omega_T} x^k_{0,j} = 1, \quad \forall k \in K && \text{(1b)}\\
& \sum_{i\in\Omega_0} x^k_{i,T} = 1, \quad \forall k \in K && \text{(1c)}\\
& \sum_{j\in\Omega_T} x^k_{ij} = y_{ik} \quad \forall i \in \Omega, k \in K && \text{(1d)}\\
& \sum_{i\in\Omega_0} x^k_{ij} = y_{jk}, \quad \forall j \in \Omega, k \in K && \text{(1e)}\\
& \sum_{k\in K} y_{ik} = 1, \quad \forall i \in \Omega, && \text{(1f)}
\end{aligned}
$$

Sequencing master problem (providing LB)

x,y →

Time allocation

$$
\begin{aligned}
& D_i + t_{ij} + p_{jk} - D_j \le M\left(1 - \sum_{k\in K} x^k_{ij}\right), \quad \forall i,j \in \Omega && \text{(1g)}\\
& D_i + p_{jk} - D_j \le M\left(1 - z_{ij}\right), \quad \forall i,j \in \Omega, l_i \ne l_j && \text{(1h)}\\
& D_j - p_{jk} - D_i \le M z_{ij}, \quad \forall i,j \in \Omega, l_i \ne l_j && \text{(1i)}\\
& y_{iu} + y_{jv} \le 1 + z_{ij} + z_{ji}, \quad \forall (i,j,v,w) \in \Theta && \text{(1j)}\\
& D_i + \Delta^{uv}_{ij} + p_{jk} - D_j \le M\left(3 - z_{ij} - y_{iu} - y_{jv}\right), \quad (i,j,v,w) \in \Theta && \text{(1k)}\\
& D_j + \Delta^{uv}_{ij} + p_{ik} - D_i \le M\left(3 - z_{ji} - y_{iu} - y_{jv}\right), \quad (i,j,v,w) \in \Theta && \text{(1l)}\\
& z_{ij} + z_{ji} = 1, \quad \forall (i,j) \in \Psi \setminus \Phi && \text{(1m)}\\
& z_{ij} = 1, z_{ji} = 0, \quad \forall (i,j) \in \Phi && \text{(1n)}\\
& r_k - D_j + t^k_{0j} + p_{jk} \le M\left(1 - x^k_{0j}\right), \quad \forall j \in \Omega, k \in K && \text{(1o)}\\
& D_j + t^k_{jT} - C_k \le M\left(1 - x^k_{jT}\right), \quad \forall j \in \Omega, k \in K && \text{(1p)}\\
& C_k \le d_k, \quad \forall k \in K && \text{(1q)}\\
& C_k \le W, \quad \forall k \in K && \text{(1r)}\\
& \sum_{i\in\Omega^F_k} y_{ik} = 0, \quad \forall k \in Q && \text{(1s)}\\
& x^k_{ij}, y_{ik}, z_{ij} \in \{0,1\}, D_i, C_k, W \ge 0, \quad \forall i,j \in \Omega, k \in K. && \text{(1t)}
\end{aligned}
$$

Time allocation sub-problem (providing upper bound UB)

← **Combinatorial cuts** ↑

Termination: LB=UB

Inspired by Sampaio et al. (2016)

3. Combinatorial Benders approach

❖ Solution of sequencing master problem

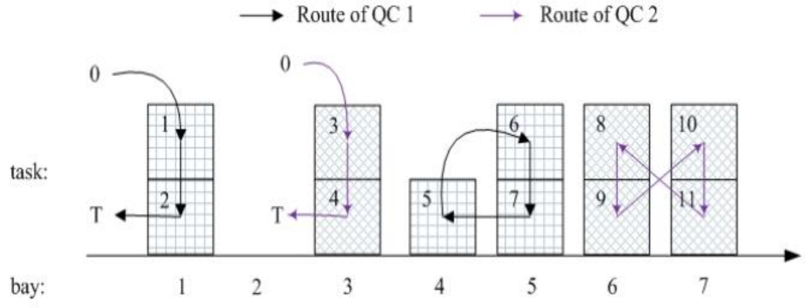
Master Problem: QC-independent operation sequences

QC k_i : 0 → i_1 → … → i-1 → i → i+1 → … → i_m → T

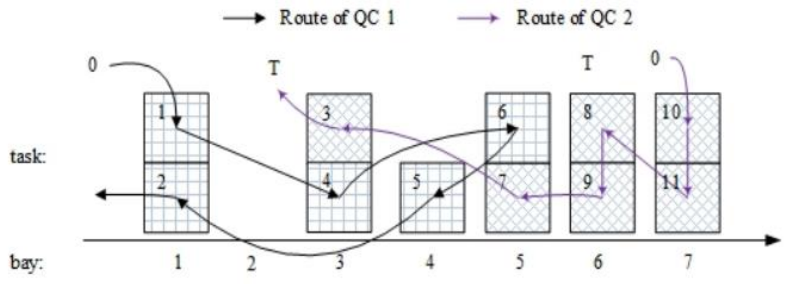
QC k_j : 0 → j_1 → … → j-1 → j → j+1 → … → j_m → T

Ignoring QC Collisions: Infeasibility and suboptimality

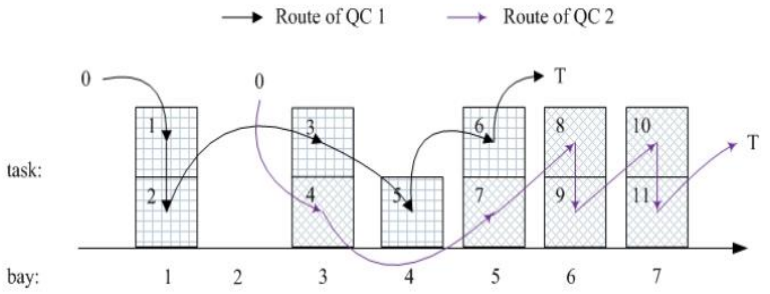
Infeasible sub-tours
e.g. 5-6-7 and 8-9-10-11



Precedence violation
e.g. (3,4) and (6,7)



QC waiting due to QC collision
e.g. task pair (3, 4)



3. Combinatorial Benders approach

- **Drawbacks of traditional combinatorial (logic-based) Benders**
  - Bad lower bound
  - Loose combinatorial cuts

$$W \geq \widetilde{W} - (\widetilde{W} - LB) \sum_{x \in C} (1 - x_{ij}^k), \quad C = \{x_{ij}^k | \widetilde{x}_{ij}^k = 1\}$$

$$\sum_{x \in C} x_{ij}^k \leq |C| - 1, \quad C = \{x_{ij}^k | \widetilde{x}_{ij}^k = 1\}$$

Subsets of C

- **Improving strategies**
  - Valid inequalities
  - Tighter combinatorial cuts
    - No-good cuts based on suboptimal subsystems
    - Infeasibility CB cuts based on infeasible subsystems
    - Optimality-property-based cuts
  - Selected multiple cuts
  - Time allocation heuristics to improve upper bound

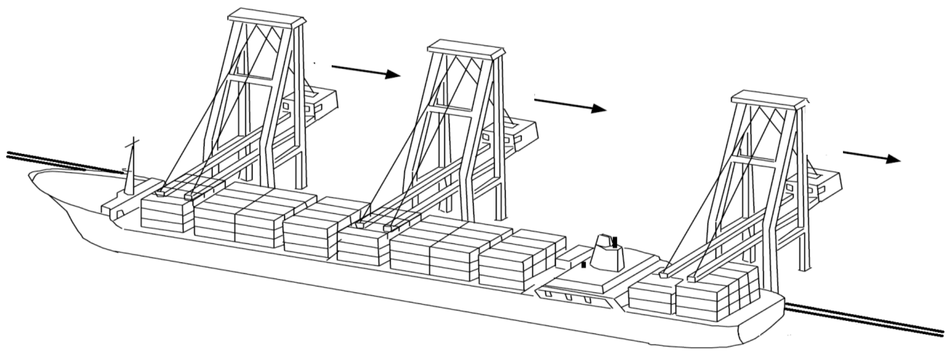
3. Combinatorial Benders approach

❖ Valid inequalities --- lower bound inequality

Theorem 1 *Given a QC allocation scheme $\widetilde{y}_{ik}$, the time allocation obtained by strategy (11) yields a valid time allocation which satisfies lower-bound inequalities (3) - (10) and leads to $\widetilde{W}_{LB} = \max_{i\in\Omega}\{D_i\}$.*

$$D_i = \max_{\sigma_i \le k \le q}\{D_{i^k_{l_i-1+(k-\sigma_i)(\delta+1)}}\} + t + \sum_{j\in\Omega: l_j = l_i, j\le i} p_{jk}\widetilde{y}_{j\sigma_i}, \forall j \in \Omega$$

- Calculate the inevitable **waiting time** due to QC collision
- Unidirectional QC movement but relaxing precedence constraints



$$W \ge \sum_{i\in\Omega} p_i y_{ik} + t\cdot\left(\sum_{h=1}^{s} h\beta_{hk} - \sum_{h=1}^{s} h\alpha_{hk}\right) + \sum_{s\in B} w_{sk}, \quad k\in K$$

$$\sum_{h=1}^{s}\sum_{i\in\Omega_h} p_i y_{ik} + \sum_{h=1}^{s} w_{hk} + t\cdot\left(s - \sum_{h=1}^{s} h\alpha_{hk}\right) + M\left(1 - \sum_{h=1}^{s'}\alpha_{hk'} + \sum_{h=1}^{s-1}\beta_{hk}\right)$$
$$\ge \sum_{h=1}^{s'}\sum_{i\in\Omega_h} p_{ik} y_{ik'} + \sum_{h=1}^{s'} w_{hk'} + t\cdot\left(s' - \sum_{h=1}^{s'} h\,\alpha_{hk'}\right),$$
$$\forall k\in K\setminus\{q\}, k' = k+1, 1\le s\le b-\delta-1, s' = s+\delta+1,$$

$$\sum_{s\in B}\alpha_{sk} = \sum_{s\in B}\beta_{sk} = 1 \quad k\in K$$

$$\sum_{s\in B, s>l_i}\alpha_{sk} + y_{ik} \le 1, \quad k\in K, i\in\Omega$$

$$\sum_{s\in B} s\beta_{sk} \ge l_i y_{ik}, \quad k\in K, i\in\Omega$$

$$\sum_{s\in B} s\alpha_{sk} \le \sum_{s\in B} s\beta_{sk} \quad k\in K$$

$$\sum_{s\in B} s\alpha_{sk} + \delta + 1 \le \sum_{s\in B} s\alpha_{s,k+1} \quad 1\le k\le q-1$$

$$\sum_{s\in B} s\beta_{sk} + \delta + 1 \le \sum_{s\in B} s\beta_{s,k+1} \quad 1\le k\le q-1$$

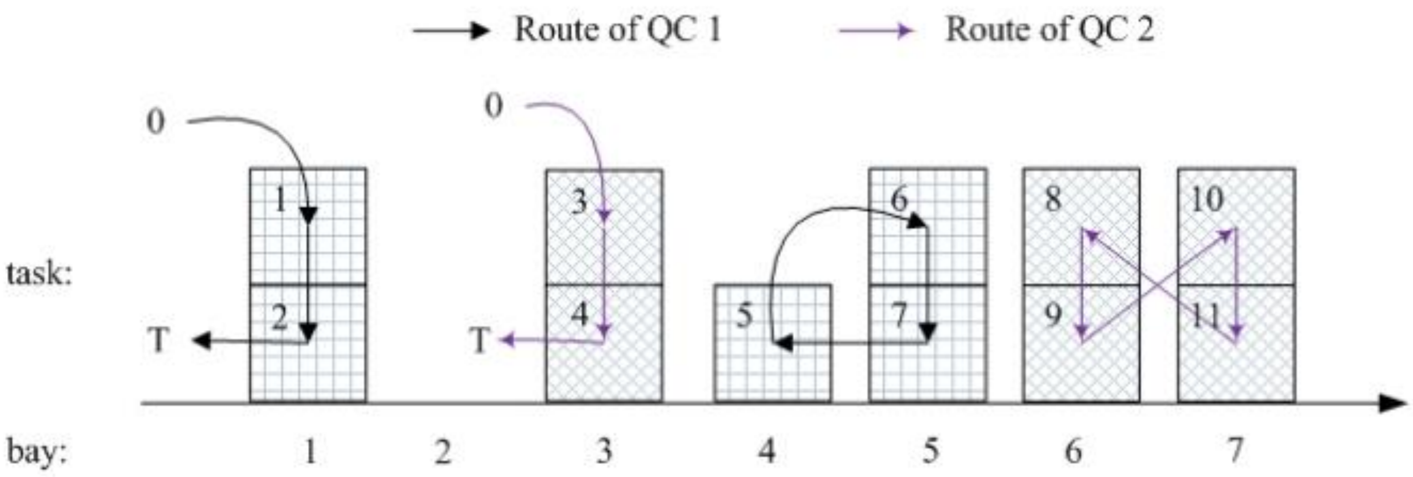
Basic idea : The total time that a given QC will stay on the left side of bay s should be no less than that of its right adjacent QC, $\forall s\in B$

3. Combinatorial Benders approach

❖ **Valid inequalities --- moving inequality**

capable of eliminating partial sub-tours

e.g. 5-6-7 and 8-9-10-11



$$\sum_{i\in\Omega}\sum_{j\in\Omega} t_{ij}x_{ij}^k + \sum_{j\in\Omega_T} t_{0j}^k x_{0j}^k + \sum_{i\in\Omega} t_{iT}^k x_{iT}^k \geq t\cdot\left(\sum_{h=1}^{s} h\beta_{hk} - \sum_{h=1}^{s} h\alpha_{hk}\right), \quad \forall k \in K.$$

QC moving time from task 0 to T

Minimum QC moving time according to bay range

Basic idea : should no less than

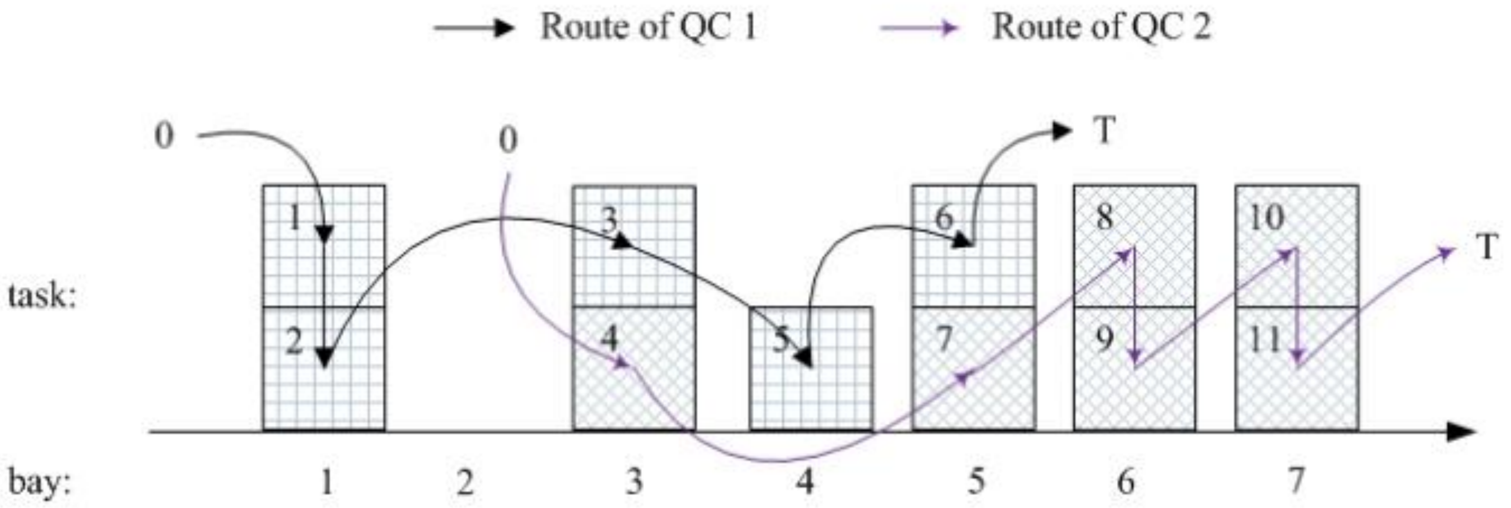
3. Combinatorial Benders approach

❖ Valid inequalities --- Precedence based makespan inequalities

task precedence pair (3,4)

QC 2: $\{0,4,7,8,9,10,11,T\}$

$\rho_4^0 = p_3 + p_4$

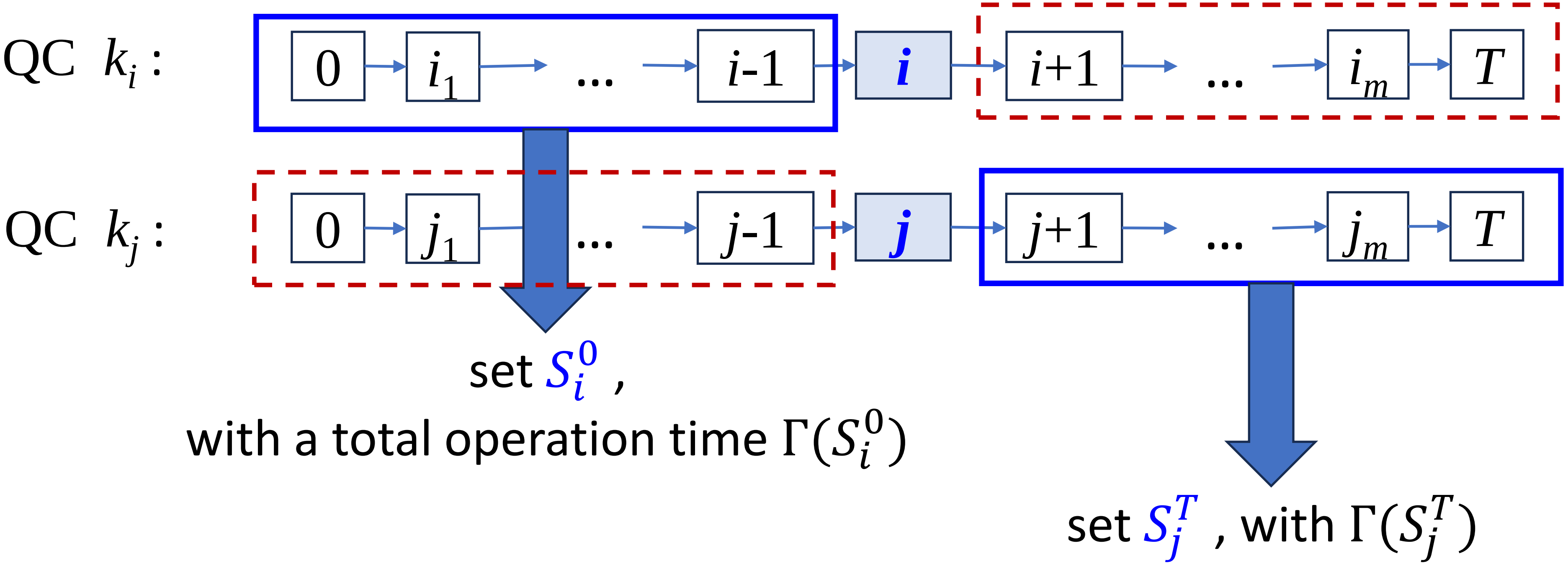


$$C_k \geq \sum_{j\in\Omega_T} \rho_j^0 x_{0j}^k + \sum_{i\in\Omega} p_i y_{ik} + \sum_{i\in\Omega}\sum_{j\in\Omega} t_{ij} x_{ij}^k + \sum_{i\in\Omega} t_{iT}^k x_{iT}^k, \quad \forall k \in Q,$$

$$W \geq \sum_{j\in\Omega_T} \rho_j^0 x_{0j}^k + \sum_{i\in\Omega} p_i y_{ik} + \sum_{i\in\Omega}\sum_{j\in\Omega} t_{ij} x_{ij}^k + \sum_{i\in\Omega} \rho_i^T x_{iT}^k, \quad \forall k \in Q.$$

3. Combinatorial Benders approach

❖ No good cuts - upper bound based cuts



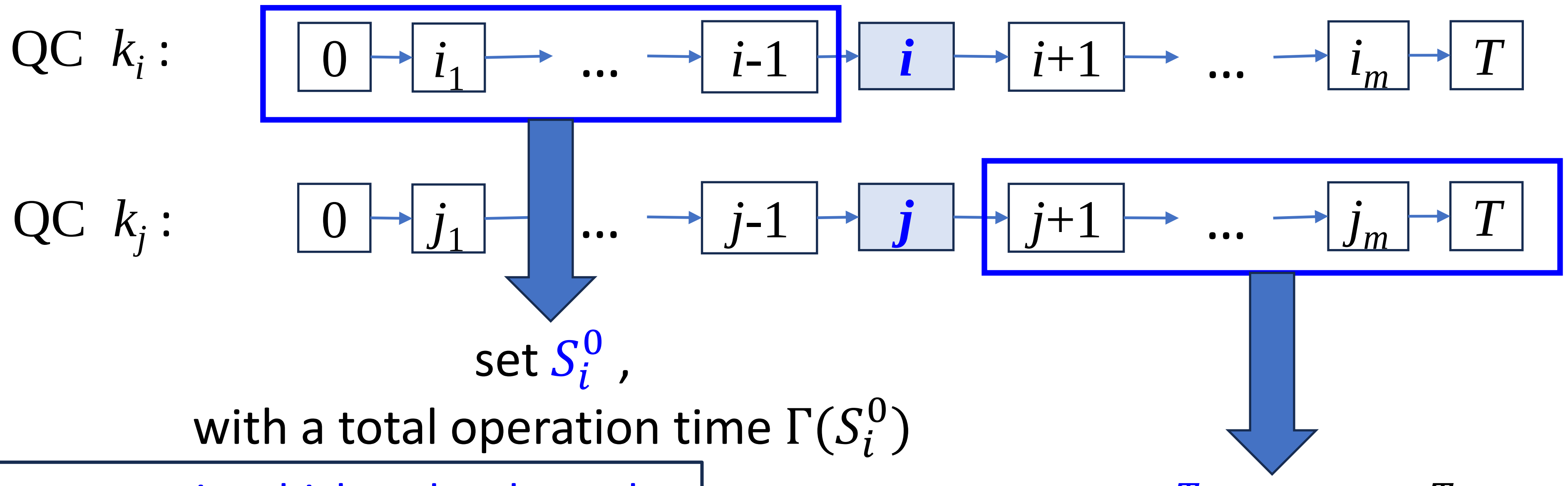
What if tasks i and j can not be performed simultaneously &

$$r_{k_i} + \Gamma(S_i^0) + p_{ik_i} + p_{jk_j} + \Gamma(S_j^T) > \min\{UB, d_{k_j}\}$$

$$r_{k_j} + \Gamma(S_j^0) + p_{jk_j} + p_{ik_i} + \Gamma(S_i^T) > \min\{UB, d_{k_i}\}$$

3. Combinatorial Benders approach

❖ **No good cuts - upper bound based cuts**



set S_i^0 ,
with a total operation time $\Gamma(S_i^0)$

set S_j^T , with $\Gamma(S_j^T)$

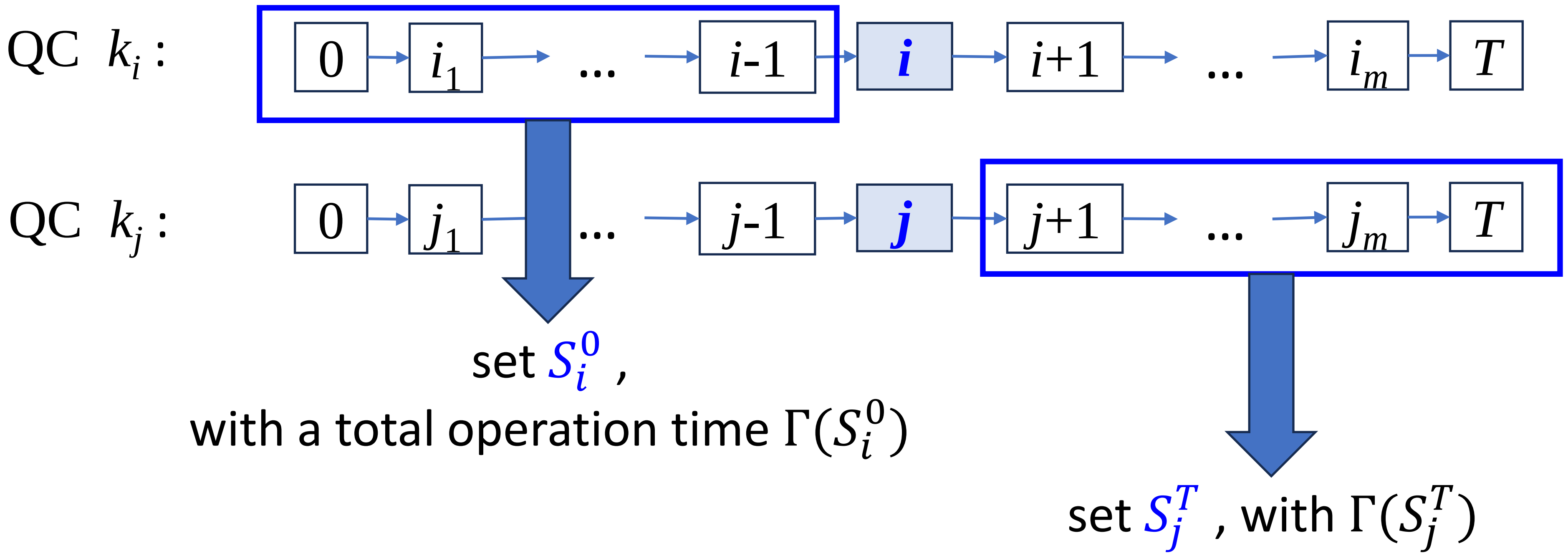
It works no matter in which order the tasks in S_i^0 and S_j^T are processed

No good cut:

$$\sum_{i'\in S_i^0}\sum_{j'\in S_i^0} x_{i',j'}^{k_i} + y_{ik_i} + \sum_{i'\in S_i^T}\sum_{j'\in S_i^T} x_{i',j'}^{k_i} + \sum_{i'\in S_j^0}\sum_{j'\in S_j^0} x_{i',j'}^{k_j} + y_{jk_j} + \sum_{i'\in S_j^T}\sum_{j'\in S_j^T} x_{i',j'}^{k_j}$$
$$\leq |S_i^0| + |S_i^T| + |S_j^0| + |S_j^T| - 3, \quad \forall (i,j,k_i,k_j) \in \Theta$$

3. Combinatorial Benders approach

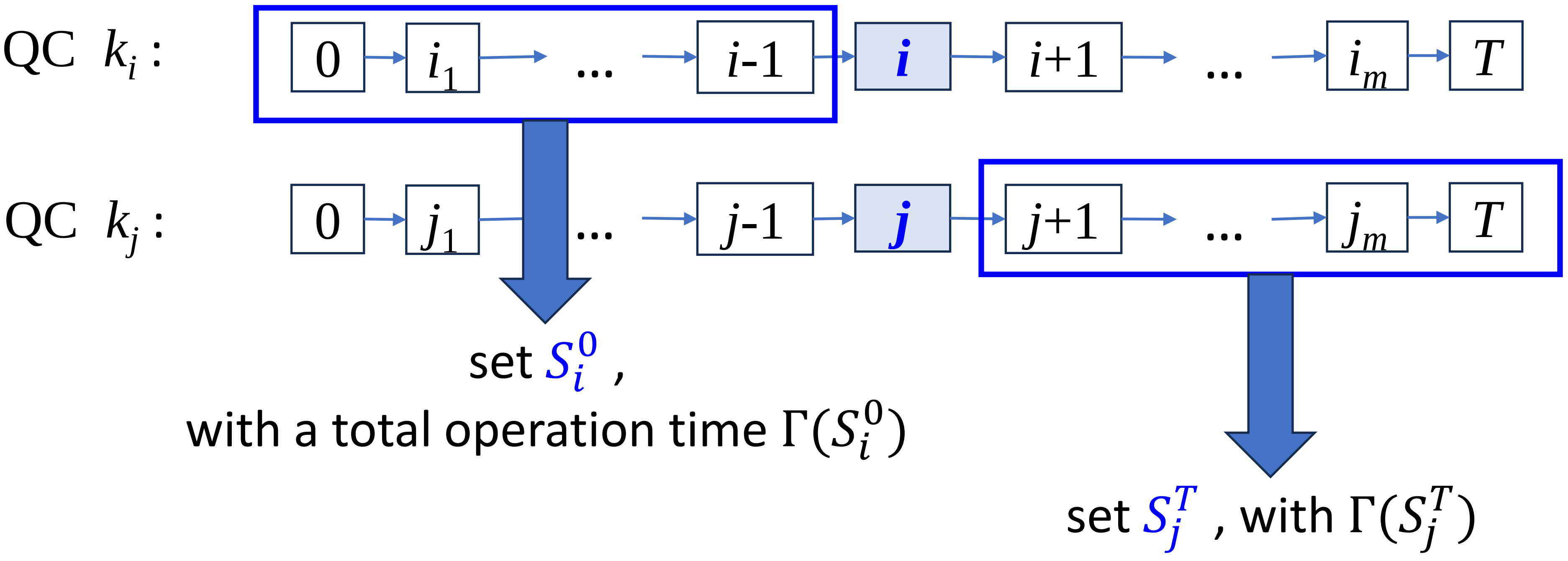
❖ **No good cuts - upper bound based cuts**



Further, what if tasks i and j satisfy the precedence relationship $(i, j) \in \Phi$

3. Combinatorial Benders approach

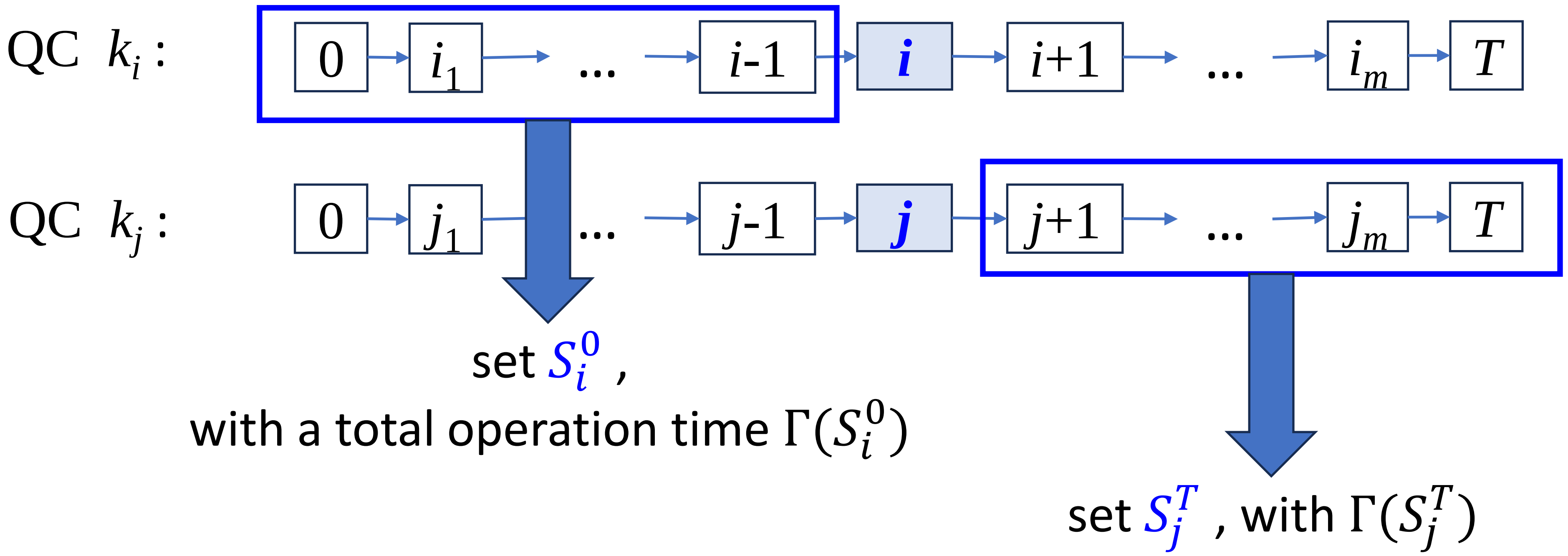
❖ **No good cuts - lifted cuts**



$$\sum_{i' \in S_i^0} \sum_{j' \in S_i^0} x_{i',j'}^{k_i} + y_{i,k_i} + \sum_{i' \in S_j^T} \sum_{j' \in S_j^T} x_{i',j'}^{k_j} + y_{j,k_j} \leq |S_i^0| + |S_j^T| - 1, \quad \forall (i,j) \in \Phi$$

3. Combinatorial Benders approach

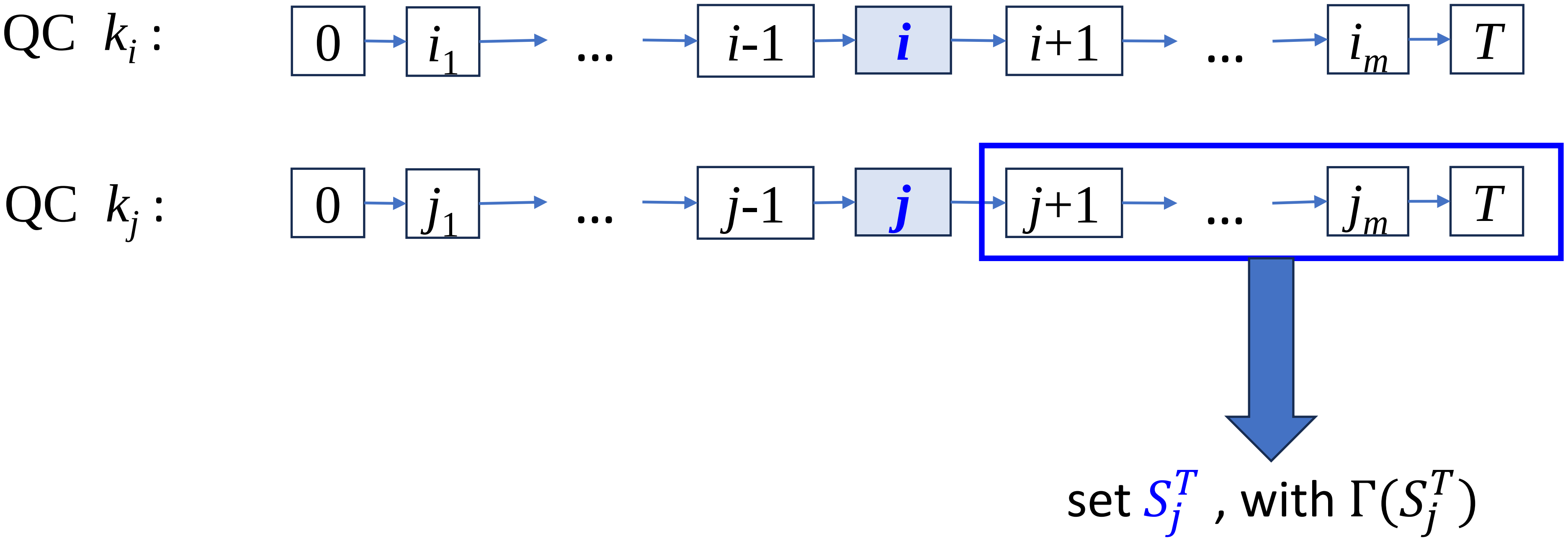
❖ No good cuts - lifted cuts



Further, what if $\sum_{i' \in \Omega, (i', j) \in \Phi} \min_{k \in Q} p_{jk} > \Gamma(S_i^0)$

3. Combinatorial Benders approach

❖ **No good cuts - lifted cuts**



$$\sum_{i' \in S_j^T} \sum_{j' \in S_j^T} x_{i',j'}^{k_j} + y_{j,k_j} \leq |S_j^T| - 1, \quad \forall j \in \Omega,\ \rho_j^0 + p_{jk} + \Gamma(S_j^T) > \min\{UB, d_{k_j}\}$$

3. Combinatorial Benders approach

❖ **No good cuts - lifted cuts**

Recall the basic no good cut involving four task sets:

$$\sum_{i' \in S_i^0} \sum_{j' \in S_i^0} x_{i',j'}^{k_i} + y_{ik_i} + \sum_{i' \in S_i^T} \sum_{j' \in S_i^T} x_{i',j'}^{k_i} + \sum_{i' \in S_j^0} \sum_{j' \in S_j^0} x_{i',j'}^{k_j} + y_{jk_j} + \sum_{i' \in S_j^T} \sum_{j' \in S_j^T} x_{i',j'}^{k_j}$$
$$\leq |S_i^0| + |S_i^T| + |S_j^0| + |S_j^T| - 3, \quad \forall (i, j, k_i, k_j) \in \Theta$$

What if $(i, j) \notin \Phi$

A second cut **lifting** routine by adding **QC collision** into consideration

$$\sum_{i' \in S_j^0} \sum_{j' \in S_j^0} x_{i',j'}^{k_j} + \sum_{j' \in S_j^0} x_{j',j}^{k_j} + \sum_{i' \in S_i^T} \sum_{j' \in S_i^T} x_{i',j'}^{k_i} + y_{ik_i} \leq |S_j^0| + |S_i^T| - 1, \quad \forall (i, j' \in S_j^0 \cup \{j\}, k_i, k_j) \in \Theta,$$

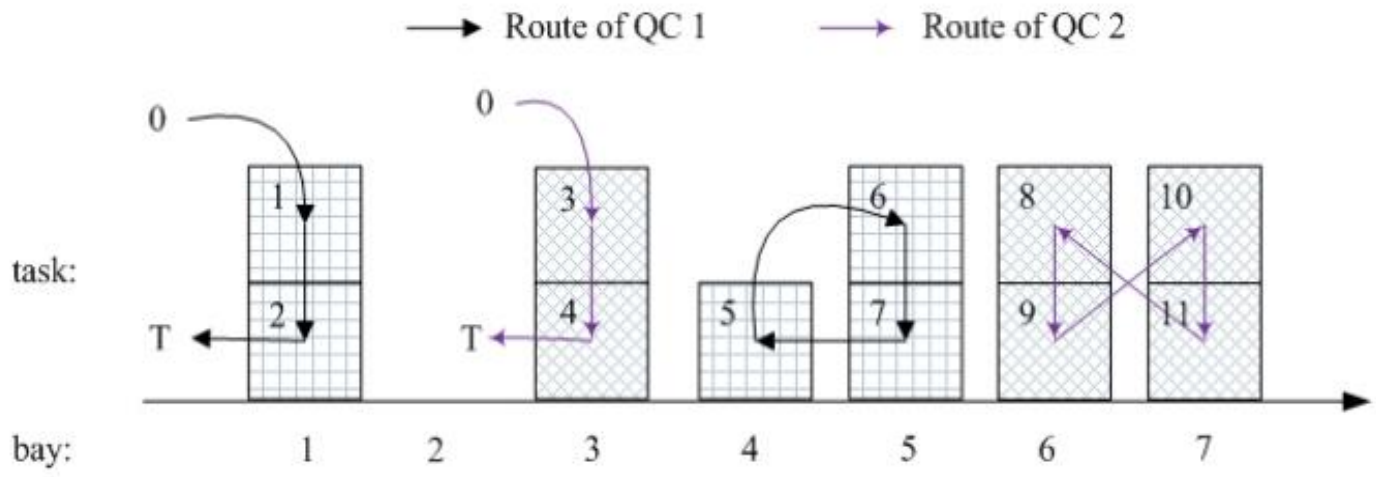
$$r_{k_j} + \Gamma(S_j^0) + p_{jk_j} + p_{ik_i} + \Gamma(S_i^T) > \min\{UB, d_{k_i}\} \qquad (29)$$

3. Combinatorial Benders approach

❖ **Infeasibility cut --- sub-tour elimination**

Let S denote the set of all tasks in a given sub-tour

e.g. 5-6-7 and 8-9-10-11



$$\sum_{i \in S} \sum_{j \in S} x_{ij}^k \leq |S| - 1,$$

$$\Rightarrow \sum_{j_1 \in S_j} \sum_{j_2 \in S_j} x_{j_1,j_2}^k + y_{ik} \leq |S_j| - 1, \quad \forall S_j, 0, j \in S_j, i \notin S_j, (i,j) \in \Phi, k \in K$$

$$\Downarrow$$

$$\sum_{j_1 \in S_k^{0T}} \sum_{j_2 \in S_k^{0T}} x_{j_1,j_2}^k + y_{ik} \leq |S_k^{0T}| - 1, \quad \forall S_k^{0T}, 0, T \in S_k^{0T}, i \notin S_k^{0T}, k \in K$$

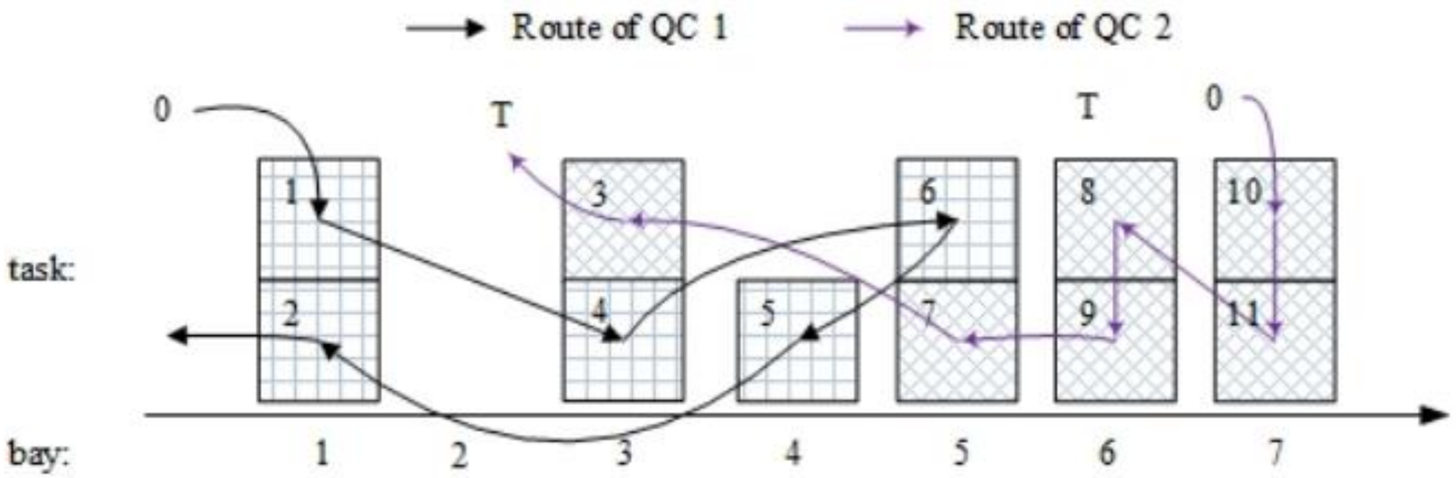
3. Combinatorial Benders approach

❖ Infeasibility cuts --- precedence violation elimination

For two task precedence pairs

$$(i_1, j_1), (i_2, j_2) \in \Phi,$$

e.g. (3,4) and (6,7)



$$\sum_{k \in K} x^k_{j_1\, i_2} + \sum_{k \in K} x^k_{j_2\, i_1} \leq 1, \quad \forall (i_1, j_1), (i_2, j_2) \in \Phi$$

Generalized to multiple precedence pairs

$$(i_1, j_1), ..., (i_m, j_m) \in \Phi$$

$$\sum_{i' \in S_{j_1}} \sum_{j' \in S_{j_1}} x^{k_1}_{i',j'} + y_{i(j_1),k_1} + ... + \sum_{i' \in S_{j_m}} \sum_{j' \in S_{j_m}} x^{k_m}_{i',j'} + y_{i(j_m),k_m}$$

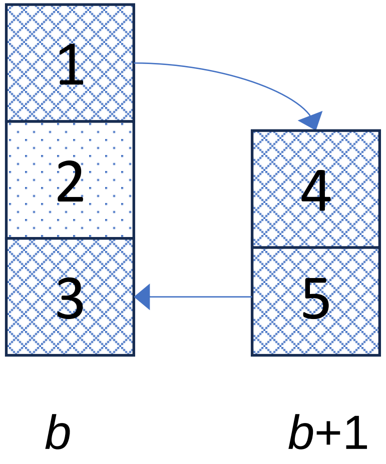
$$\leq |S_{j_1}| + ... + |S_{j_m}| - 1$$

3. Combinatorial Benders approach

❖ Infeasibility cuts --- safe-margin violation elimination

Idea: an operation sequence of QC *k* which gives other QCs no chance to operate a task due to safe-margin and precedence restrictions

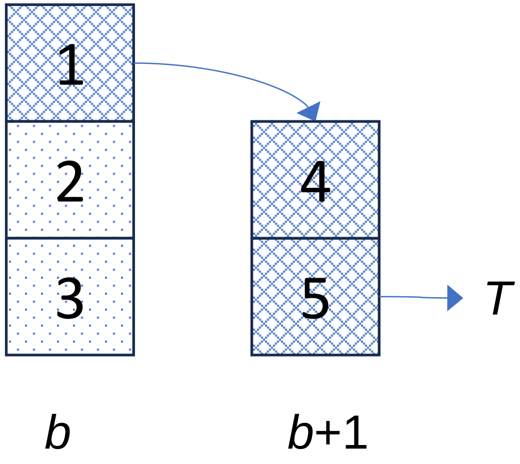
e.g. task precedence pair (1,2), (2,3), safe margin equals to 1



Cuts:

$$\sum_{i'\in\{i\}\cup S_{ij_1}} \sum_{j'\in\{i\}\cup S_{ij_1}} x_{i'j'}^{k_i} + y_{jk_j} \le |S_{ij_1}|, \quad \forall (i,j),(j,j_1)\in\Phi, (i',j,k_i,k_j)\in\Delta, i'\in S_{ij_1}$$

$$\sum_{j'\in S_i^T\setminus\{T\}} x_{ij'}^{k_i} + \sum_{i'\in S_i^T} \sum_{j'\in S_i^T} x_{i'j'}^{k_i} + y_{jk_j} \le |S_i^T|, \quad \forall (i,j)\in\Phi, (i',j,k_i,k_j)\in\Delta, i'\in S_i^T$$



3. Combinatorial Benders approach

❖ Selected multiple cuts

- Main idea of generating multiple cuts: change the elements in S_i^0 and S_j^T, can be easily realized by neighborhood search
- Selecting the cuts: a set of subset-based cuts, each of which covers as many suboptimal solutions as possible

$$\sum_{i' \in S_i^0} \sum_{j' \in S_i^0} x_{i',j'}^{k_i} + y_{i,k_i} + \sum_{i' \in S_j^T} \sum_{j' \in S_j^T} x_{i',j'}^{k_j} + y_{j,k_j} \leq |S_i^0| + |S_j^T| - 1, \quad \forall (i,j) \in \Phi$$

Let $\widetilde{S}_i^0$ and $\widetilde{S}_j^T$ be subsets of S_i^0 and S_j^T, respectively.

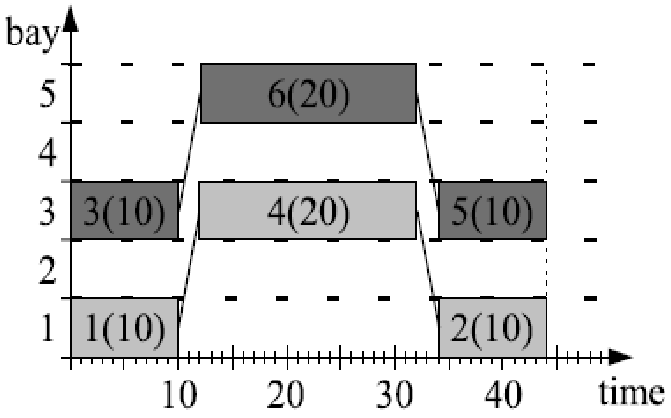
And $\Gamma(\widetilde{S}_i^0) + p_{ik} + p_{jk} + \Gamma(\widetilde{S}_j^T) > UB,$

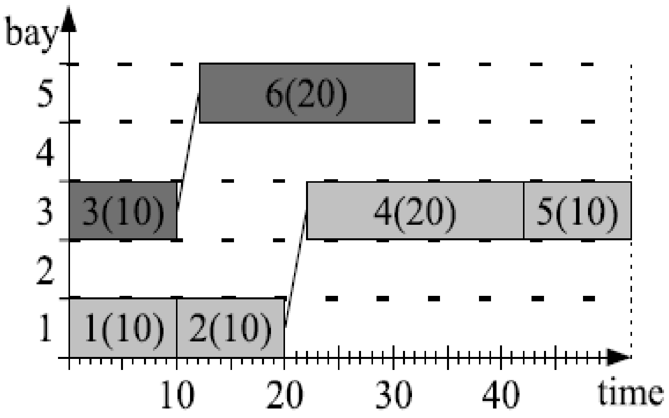
$$\sum_{i' \in \widetilde{S}_i^0} \sum_{j' \in \widetilde{S}_i^0} x_{i',j'}^{k_i} + y_{i,k_i} + \sum_{i' \in \widetilde{S}_j^T} \sum_{j' \in \widetilde{S}_j^T} x_{i',j'}^{k_j} + y_{j,k_j} \leq |S_i^0| + |S_j^T| - 1, \quad \forall (i,j) \in \Phi$$

3. Combinatorial Benders approach

❖ **Time allocation heuristics for improving upper bound**

Recall that assignment determines sequence in uni- and bi-directional QCSP





Step 1: check if assignment meets the requirement of uni- and bi-directional QCSP

Step 2: If yes, solve uni- and bi-directional QCSP under given task-QC assignments

Step 3: Possibly update upper bound UB.

4. Computational Results

❖ Benchmark instances

Table 1 Benchmark instances from the literature

Source	Set	#inst.	n	b	q	Main features
Kim and Park (2004)	A	10	10	10	2	Number of tasks always equals the number of bays. Assignment of tasks to bays and processing times of the tasks are drawn from uniform distributions. Constant safety margin.
	B	10	15	15	2	
	C	10	20	20	3	
	D	10	25	25	3	
	E	10	30	30	4	
	F	10	35	35	4	
	G	10	40	40	5	
	H	10	45	45	5	
	I	10	50	50	6	
Meisel and Bierwirth (2011)	A1	70	$[10, 15, ..., 40]$	10	2	Bay capacity of 200 containers. Handling rate is equal to 0.5. Handling volume is composed of 10-40 container groups.
	B1	60	$[45, 50, ..., 70]$	15	4	Bay capacity of 400 containers. Handling rate is equal to 0.5. Handling volume is composed of 45-70 container groups.
	C1	60	$[75, 80, ..., 100]$	20	6	Bay capacity of 600 containers. Handling rate is equal to 0.5. Handling volume is composed of 75-100 container groups.
	D1	60	50	15	4	Different spatial distributions of container groups (uniform and Gaussian). Handling rate ranges in $\{0.2, 0.8\}$.
	E1	50	50	15	4	Varying precedence densities. Container groups are uniformly distributed.
	F1	50	50	15	$[2, 3, ..., 6]$	Identically structured task data. QC number is ranging from 2 to 6.
	G1	50	50	15	4	Identically structured task data. Increasing safety requirements.

4. Computational Results

			BidBD			*SBD*			*CBD* − 1			*CBD*		
Source	Set	#inst.	#opt	%lb1	t	#opt	%lb1	t	#opt	%lb1	t	#opt	%lb1	t
Kim and Park (2004)	A	10	10	100	0.29	8	97.58	45.93	10	100	1.47	10	100	0.75
	B	10	9	99.86	0.41	8	98.69	91.10	10	99.86	19.57	10	99.86	1.97
	C	10	4	99.72	1.23	2	97.32	177.40	9	99.80	94.22	10	99.80	13.77
	D	10	3	99.63	2.06	0	98.39	-	7	99.63	108.27	10	99.63	88.01
	E	10	7	99.84	6.15	0	98.96	-	5	99.84	1013.65	10	99.84	590.22
	F	10	6	99.83	9.96	0	98.72	-	2	99.83	2539.20	10	99.83	897.08
	G	10	5	99.72	72.69	0	98.59	-	0	99.72	-	7	99.72	1214.96
	H	10	4	99.73	87.83	0	98.26	-	0	99.73	-	4	99.73	1539.46
	I	10	4	99.63	533.10	0	98.38	-	0	99.63	-	0	99.63	-
Meisel and Bierwirth (2011)	A1	70	29	99.79	0.91	19	99.48	420.54	69	99.87	40.33	70	99.87	9.45
	B1	60	23	99.78	5.19	0	97.71	-	27	99.86	973.67	43	99.86	385.79
	C1	60	43	99.90	37.82	0	97.23	-	0	99.93	-	44	99.93	685.65
	D1	60	25	99.79	13.15	0	98.23	-	32	99.89	415.40	43	99.89	660.22
	E1	50	15	99.69	5.04	0	97.89	-	17	99.76	529.66	31	99.76	322.21
	F1	50	32	99.89	2.22	0	98.66	-	27	99.93	434.76	43	99.93	47.79
	G1	50	33	99.88	8.62	0	98.31	-	31	99.93	379.73	40	99.93	167.22
		490	254			37			246			385		

Number of instances optimally solved within 1 hour

5. Conclusion

- The most efficient exact algorithm to solve QCSP known so far.
- Future work 1 : keep generalizing formulation and algorithm to cover more QCSP variants.
- Future work 2: more efficient strategies for selecting multiple cuts.
- Future work 3: Robust method for uncertain QCSP.

Thanks for your attention

Q&A

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