Train Scheduling: models, decomposition methods and practice

Carlo Mannino
Dept. of mathematics and cybernetics, SINTEF
Dept. of mathematics, University of Oslo

www.schedulingseminar.com

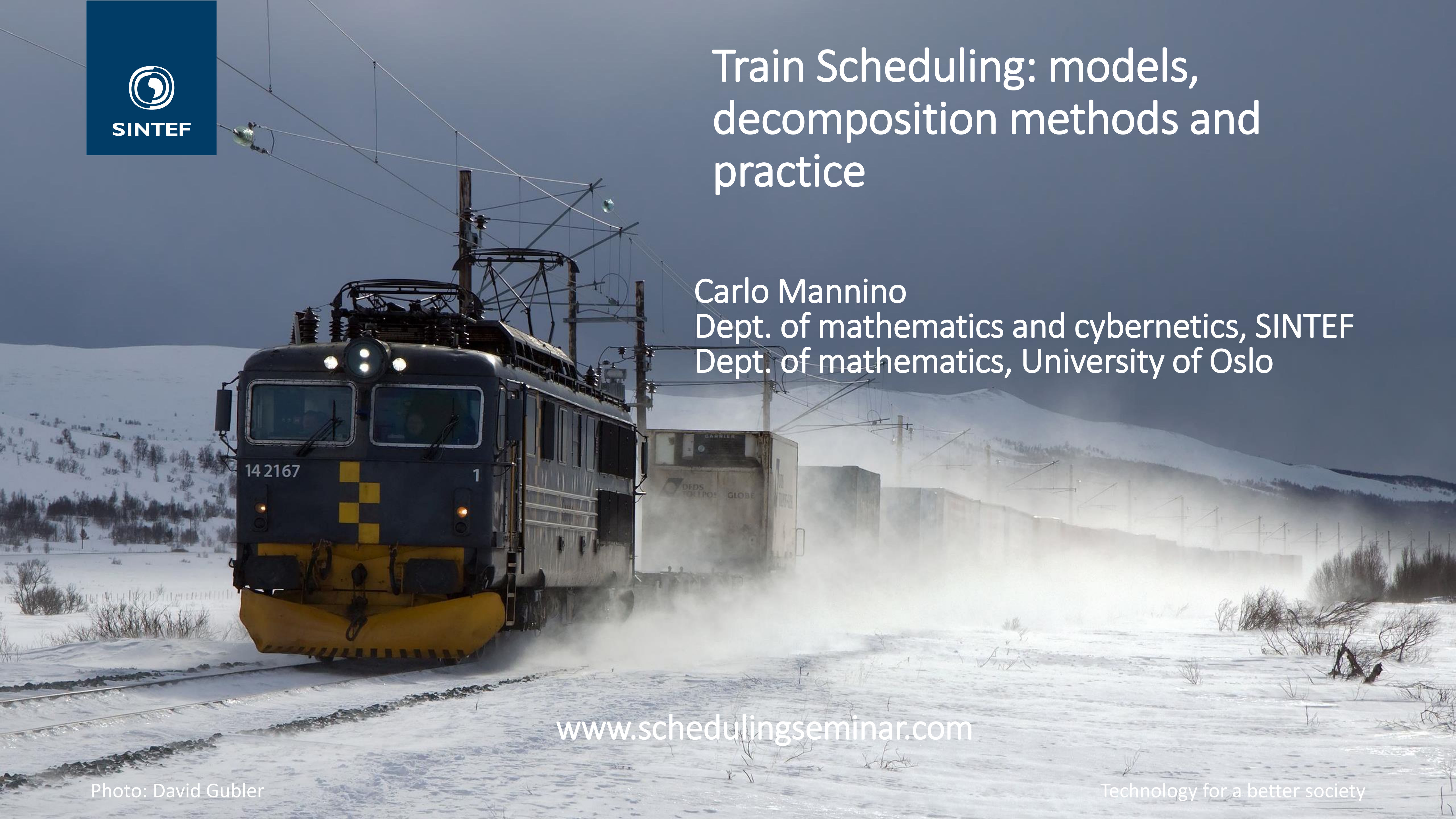
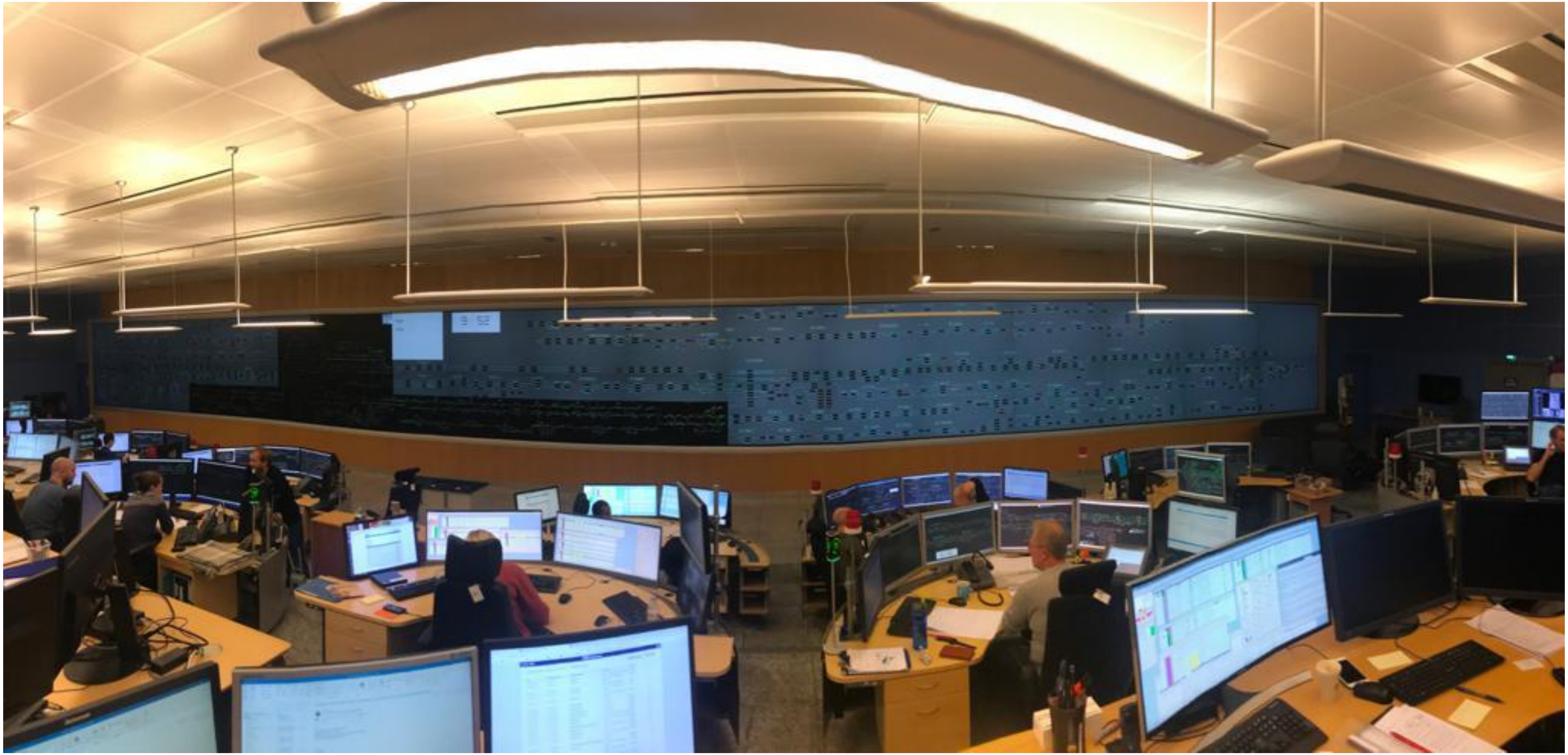
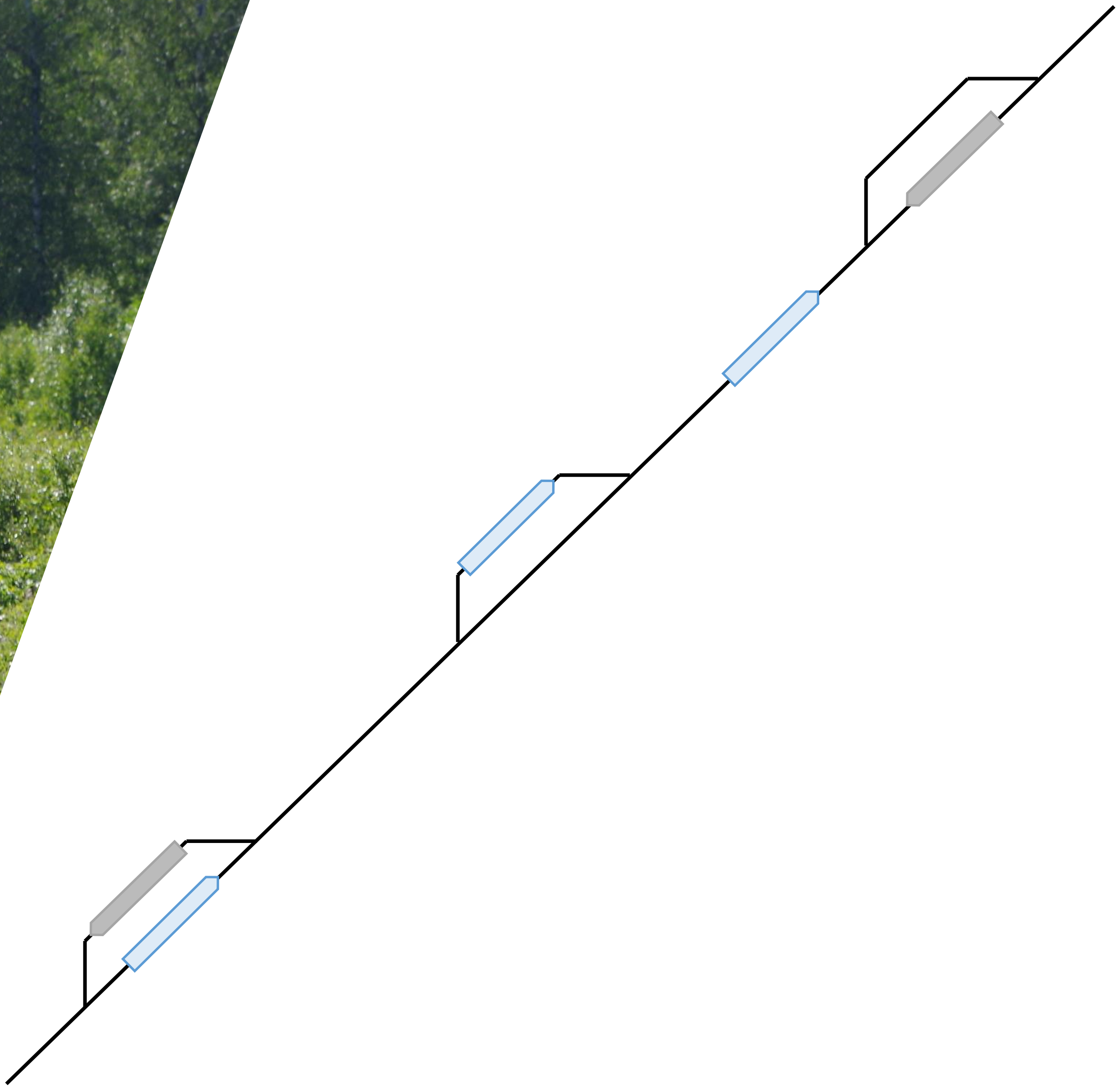


Photo: David Gubler

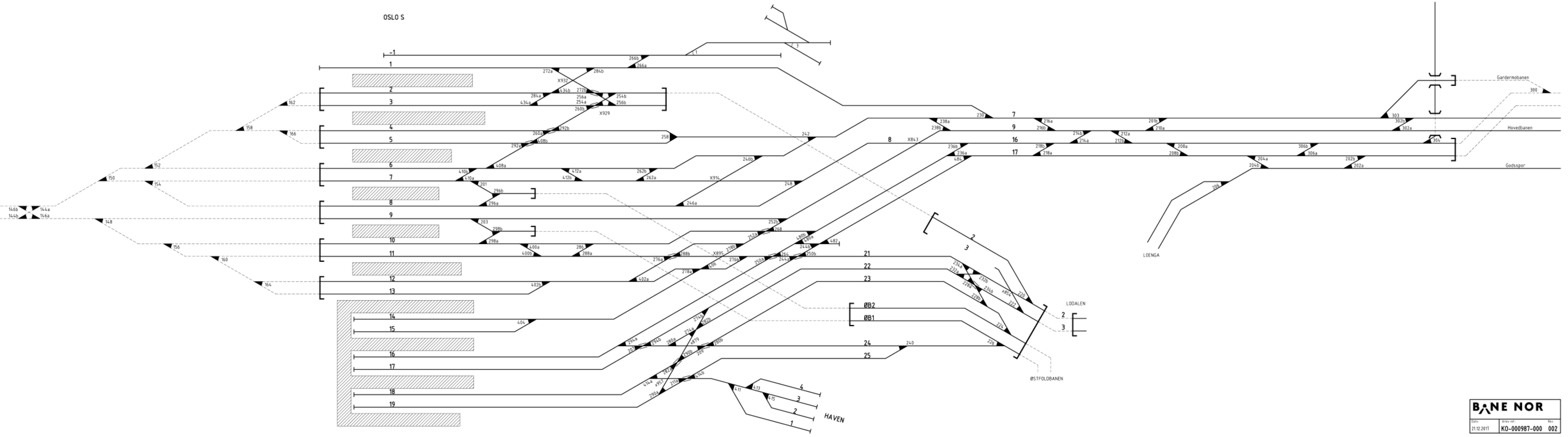
Dispatchers at work



SINTEF



Oslo Central Station

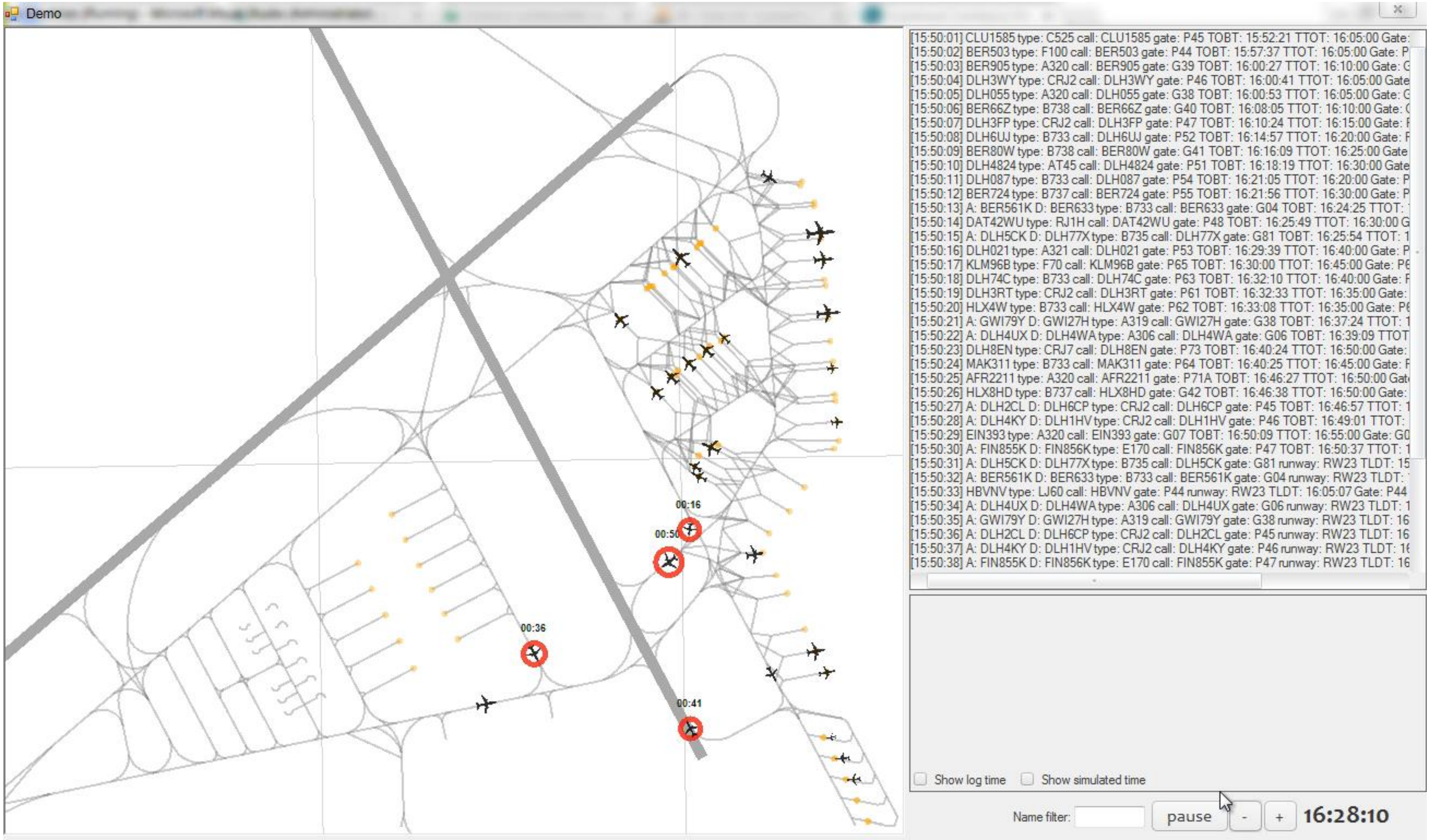


Train Scheduling: Two basic versions

- ❑ Operational (real-time): train rescheduling (dispatching)
- ❑ Tactical/Strategical: train timetabling
- ❑ Train scheduling: a job-shop scheduling problem
- ❑ Job-shop scheduling problem arising in other applications

SINTEF

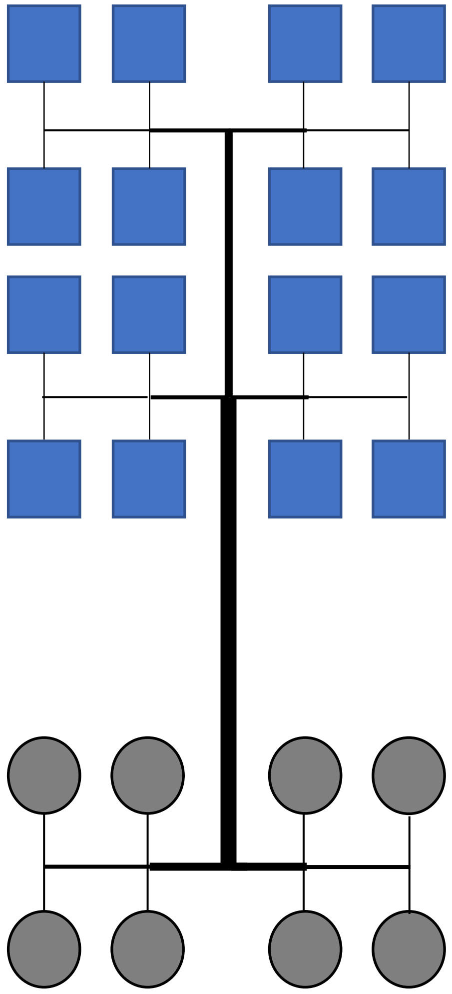

Eurowings
wizzair.com
Ukraine International
vueling.com
Raiffeisen
Eurowings
Raiffeisen
838
ROJANA



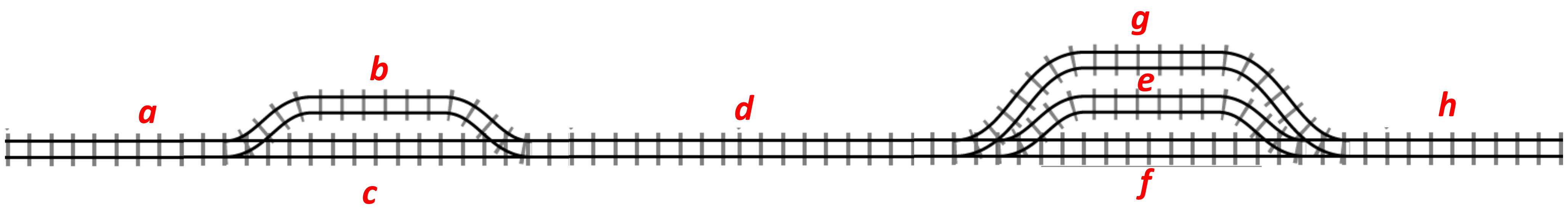
Demo
[15:50:01] CLU1585 type: C525 call: CLU1585 gate: P45 TOBT: 15:52:21 TTOT: 16:05:00 Gate:
[15:50:02] BER503 type: F100 call: BER503 gate: P44 TOBT: 15:57:37 TTOT: 16:05:00 Gate: P
[15:50:03] BER905 type: A320 call: BER905 gate: G39 TOBT: 16:00:27 TTOT: 16:10:00 Gate: G
[15:50:04] DLH3WY type: CRJ2 call: DLH3WY gate: P46 TOBT: 16:00:41 TTOT: 16:05:00 Gate
[15:50:05] DLH055 type: A320 call: DLH055 gate: G38 TOBT: 16:00:53 TTOT: 16:05:00 Gate: G
[15:50:06] BER66Z type: B738 call: BER66Z gate: G40 TOBT: 16:08:05 TTOT: 16:10:00 Gate: (
[15:50:07] DLH3FP type: CRJ2 call: DLH3FP gate: P47 TOBT: 16:10:24 TTOT: 16:15:00 Gate: F
[15:50:08] DLH6UJ type: B733 call: DLH6UJ gate: P52 TOBT: 16:14:57 TTOT: 16:20:00 Gate: F
[15:50:09] BER80W type: B738 call: BER80W gate: G41 TOBT: 16:16:09 TTOT: 16:25:00 Gate
[15:50:10] DLH4824 type: AT45 call: DLH4824 gate: P51 TOBT: 16:18:19 TTOT: 16:30:00 Gate
[15:50:11] DLH087 type: B733 call: DLH087 gate: P54 TOBT: 16:21:05 TTOT: 16:20:00 Gate: P
[15:50:12] BER724 type: B737 call: BER724 gate: P55 TOBT: 16:21:56 TTOT: 16:30:00 Gate: P
[15:50:13] A: BER561K D: BER633 type: B733 call: BER633 gate: G04 TOBT: 16:24:25 TTOT:
[15:50:14] DAT42WU type: RJ1H call: DAT42WU gate: P48 TOBT: 16:25:49 TTOT: 16:30:00 G
[15:50:15] A: DLH5CK D: DLH77X type: B735 call: DLH77X gate: G81 TOBT: 16:25:54 TTOT: 1
[15:50:16] DLH021 type: A321 call: DLH021 gate: P53 TOBT: 16:29:39 TTOT: 16:40:00 Gate: P
[15:50:17] KLM96B type: F70 call: KLM96B gate: P65 TOBT: 16:30:00 TTOT: 16:45:00 Gate: P6
[15:50:18] DLH74C type: B733 call: DLH74C gate: P63 TOBT: 16:32:10 TTOT: 16:40:00 Gate: F
[15:50:19] DLH3RT type: CRJ2 call: DLH3RT gate: P61 TOBT: 16:32:33 TTOT: 16:35:00 Gate:
[15:50:20] HLX4W type: B733 call: HLX4W gate: P62 TOBT: 16:33:08 TTOT: 16:35:00 Gate: P6
[15:50:21] A: GWI79Y D: GWI27H type: A319 call: GWI27H gate: G38 TOBT: 16:37:24 TTOT: 1
[15:50:22] A: DLH4UX D: DLH4WA type: A306 call: DLH4WA gate: G06 TOBT: 16:39:09 TTOT
[15:50:23] DLH8EN type: CRJ7 call: DLH8EN gate: P73 TOBT: 16:40:24 TTOT: 16:50:00 Gate:
[15:50:24] MAK311 type: B733 call: MAK311 gate: P64 TOBT: 16:40:25 TTOT: 16:45:00 Gate: F
[15:50:25] AFR2211 type: A320 call: AFR2211 gate: P71A TOBT: 16:46:27 TTOT: 16:50:00 Gat
[15:50:26] HLX8HD type: B737 call: HLX8HD gate: G42 TOBT: 16:46:38 TTOT: 16:50:00 Gate:
[15:50:27] A: DLH2CL D: DLH6CP type: CRJ2 call: DLH6CP gate: P45 TOBT: 16:46:57 TTOT: 1
[15:50:28] A: DLH4KY D: DLH1HV type: CRJ2 call: DLH1HV gate: P46 TOBT: 16:49:01 TTOT:
[15:50:29] EIN393 type: A320 call: EIN393 gate: G07 TOBT: 16:50:09 TTOT: 16:55:00 Gate: G0
[15:50:30] A: FIN855K D: FIN856K type: E170 call: FIN856K gate: P47 TOBT: 16:50:37 TTOT: 1
[15:50:31] A: DLH5CK D: DLH77X type: B735 call: DLH5CK gate: G81 runway: RW23 TLDT: 15
[15:50:32] A: BER561K D: BER633 type: B733 call: BER561K gate: G04 runway: RW23 TLDT:
[15:50:33] HBVNV type: LJ60 call: HBVNV gate: P44 runway: RW23 TLDT: 16:05:07 Gate: P44
[15:50:34] A: DLH4UX D: DLH4WA type: A306 call: DLH4UX gate: G06 runway: RW23 TLDT: 1
[15:50:35] A: GWI79Y D: GWI27H type: A319 call: GWI79Y gate: G38 runway: RW23 TLDT: 16
[15:50:36] A: DLH2CL D: DLH6CP type: CRJ2 call: DLH2CL gate: P45 runway: RW23 TLDT: 16
[15:50:37] A: DLH4KY D: DLH1HV type: CRJ2 call: DLH4KY gate: P46 runway: RW23 TLDT: 16
[15:50:38] A: FIN855K D: FIN856K type: E170 call: FIN855K gate: P47 runway: RW23 TLDT: 16
00:16
00:50
00:36
00:41
Show log time
Show simulated time
Name filter:
pause
-
+
16:28:10

SINTEF


Photo: Chris Reynolds

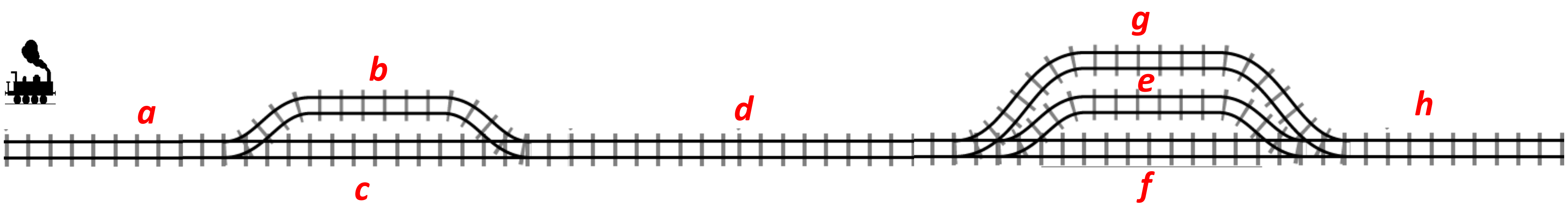


Network representation



- The tracks of the railway are segmented into elementary "blocks"
- Each block can accommodate at most one train at a time

Modelling train movement

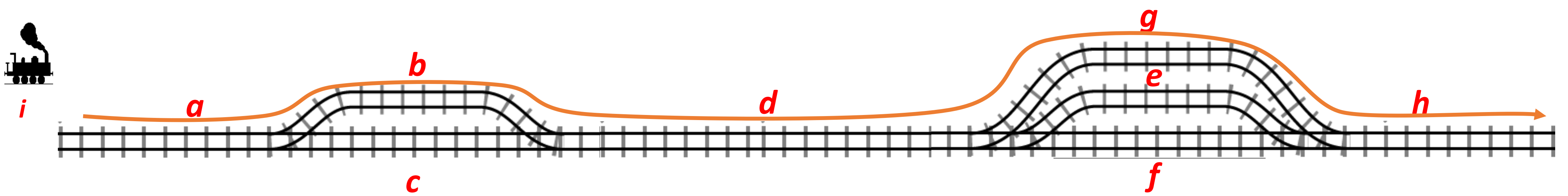


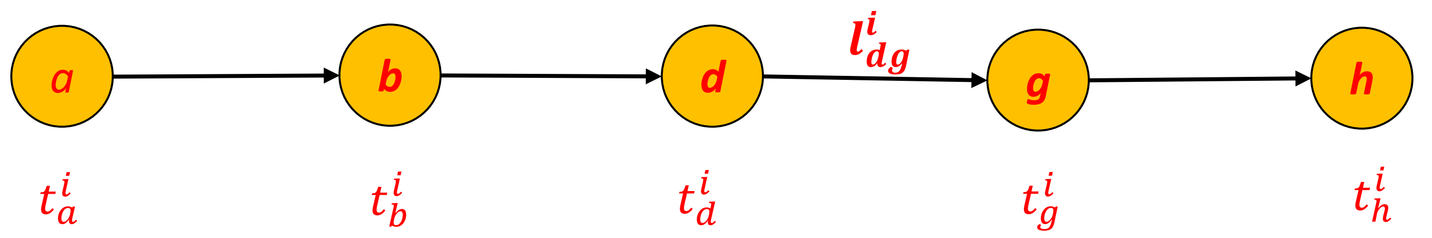
- A train runs through a sequence of blocks (its *route*)
- t_q^i is the time train i enters block q (*schedule variable*)
- If t_u is the time the train enters a block, and t_v when it enters next one, then

$$t_v - t_u \geq l_{uv},$$

where l_{uv} is the minimum running for the train through the block

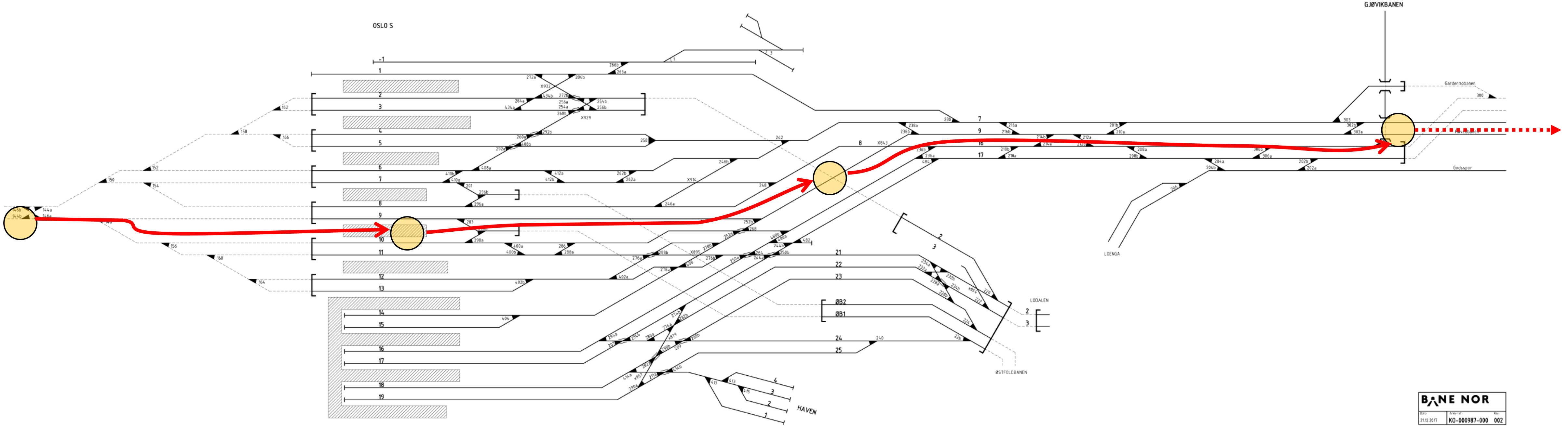
The route graph



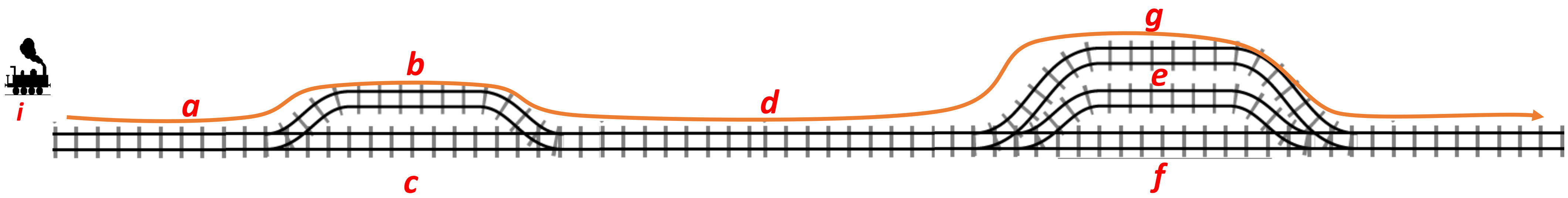


- ❑ The train movement represented by *route graph*
- ❑ Nodes correspond to (the event) *entering a block section.*
- ❑ Edges represent time precedence constraints $t^i_g - t^i_d \geq l^i_{dg}$

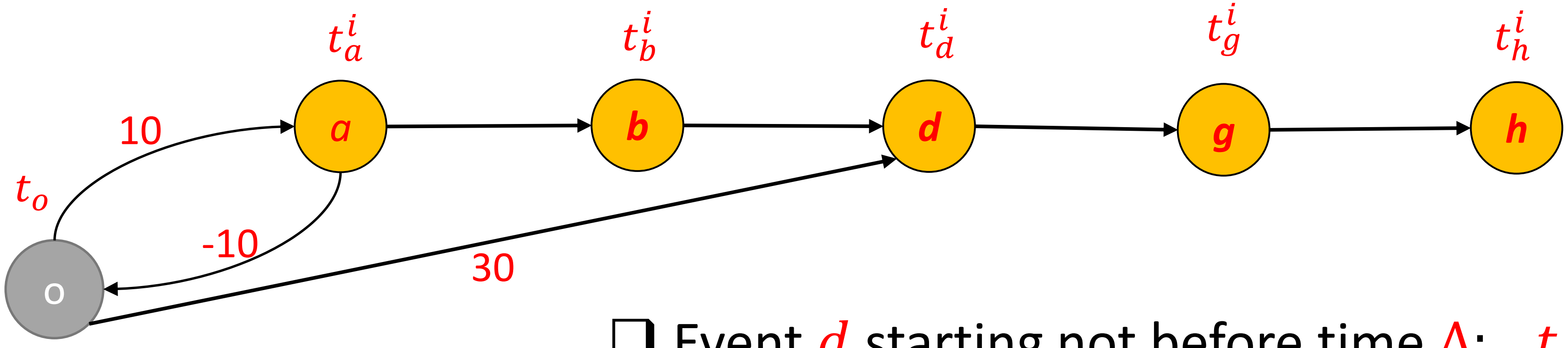
A route graph in Oslo S



The time origin



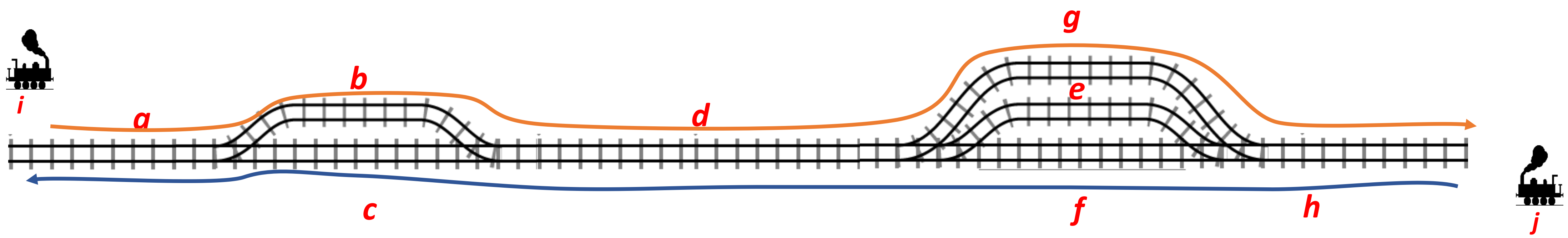
- We add a node o representing the start t_o of the planning horizon



Timetable constraints/edges

- Event d starting <u>not before</u> time Δ: $t_d - t_o \geq 30$
- Event a starting <u>at</u> time Δ: $t_a - t_o = 10 \rightarrow \begin{cases} t_a - t_o \geq 10 \\ t_o - t_a \geq -10 \end{cases}$

(*potential*) Conflicts



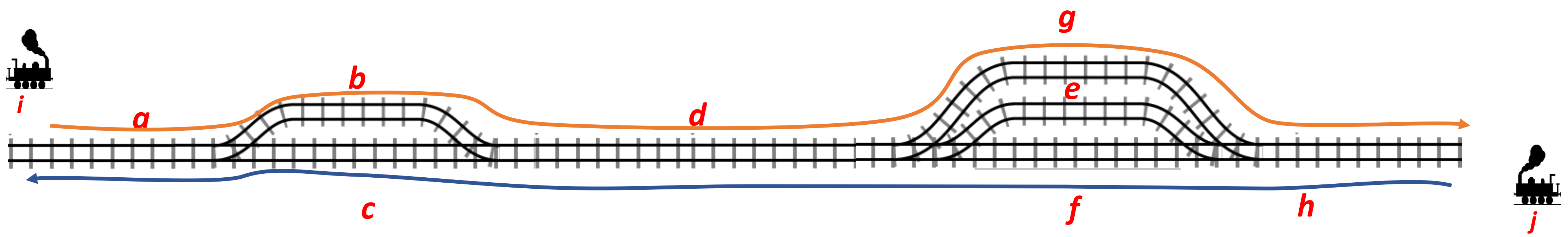
- Trains compete for the same blocks

- Either train *i* enters block *g* before *j* enters *d*: $t_d^j - t_g^i \geq 0$

- Or train *j* enters block *c* before *i* enters *d*: $t_d^i - t_c^j \geq 0$

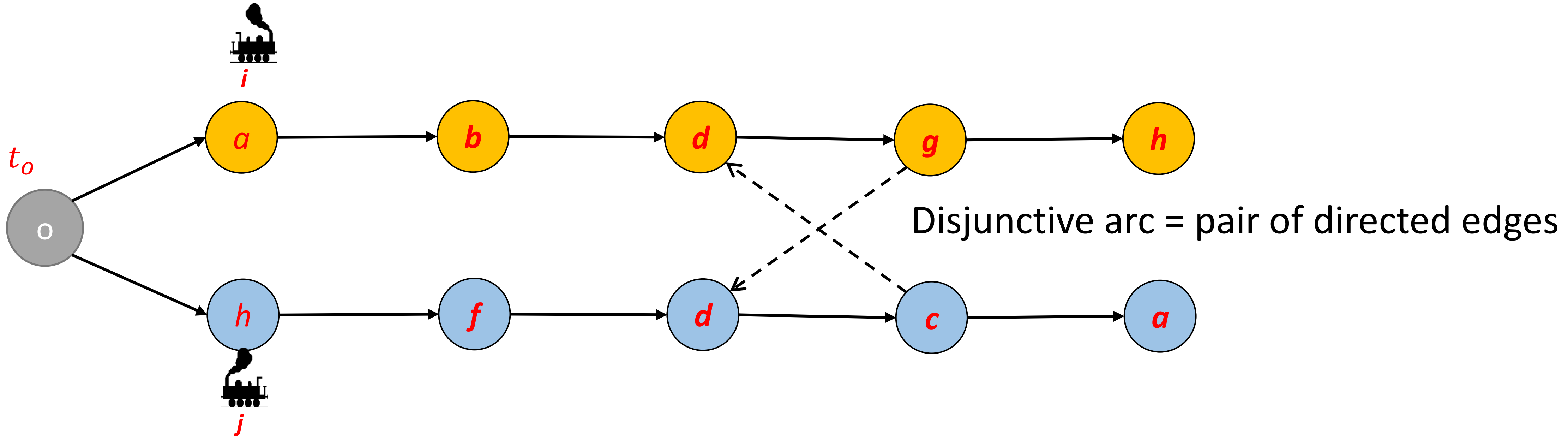
$$t_d^j - t_g^i \geq 0 \bigvee t_d^i - t_c^j \geq 0$$ Disjunctive constraint

Disjunctive arc

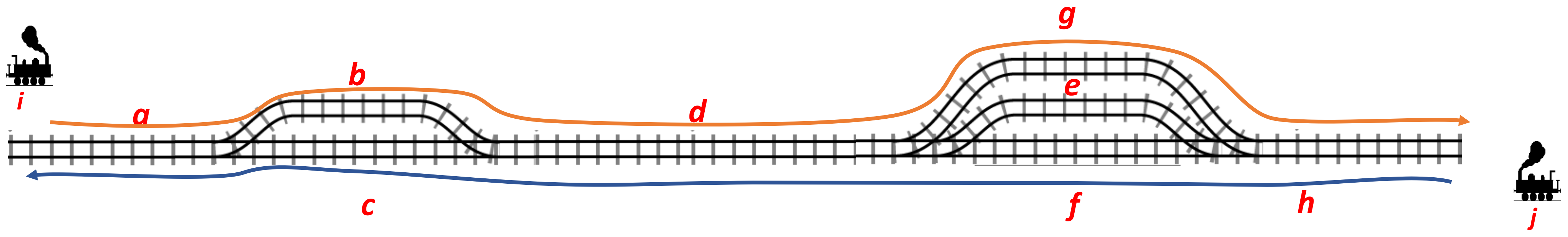


$$t_d^j - t_g^i \geq 0 \bigvee t_d^i - t_c^j \geq 0$$

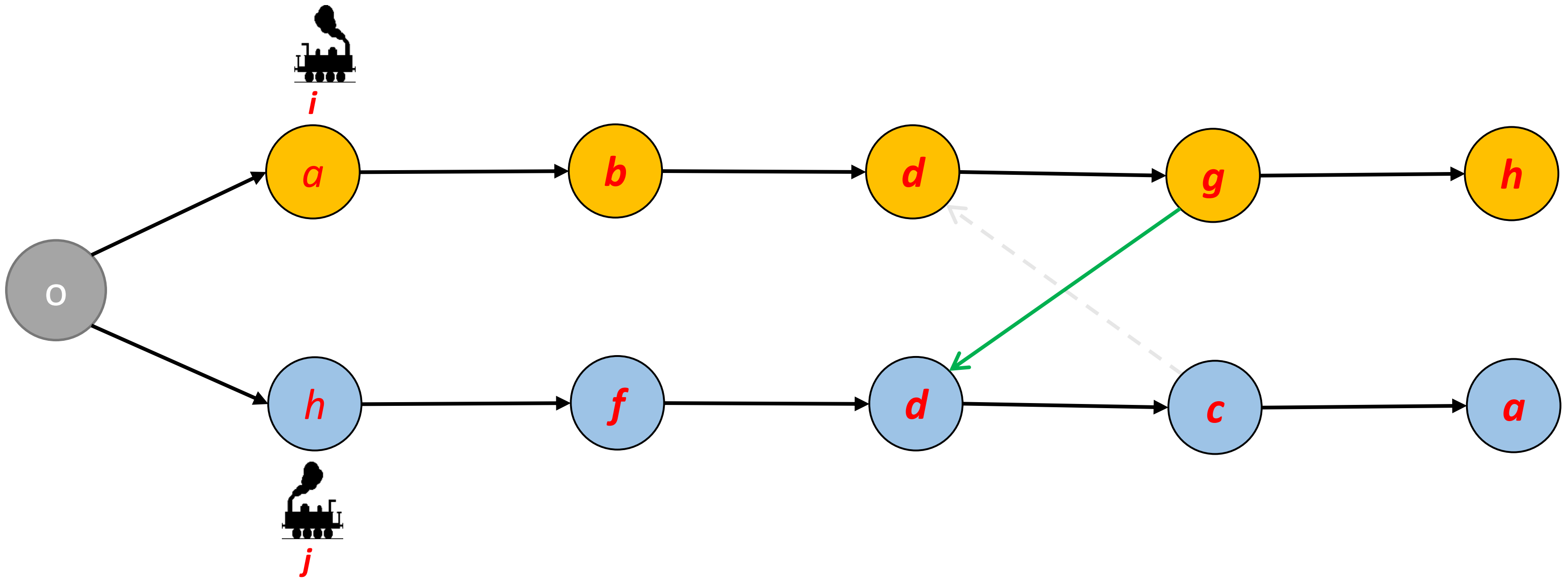
Disjunctive constraint



"Solving" Conflicts



- ❑ Solving a conflict means deciding which term in $t_d^j - t_g^i \geq 0$ **OR** $t_d^i - t_c^j \geq 0$ to satisfy



train i goes first

$t_d^j - t_g^i \geq 0$

Train scheduling problem

- Network N, set trains I (with current position) and a wanted timetable T.
- T_s^i is the arrival time of train i at station s.

WANT

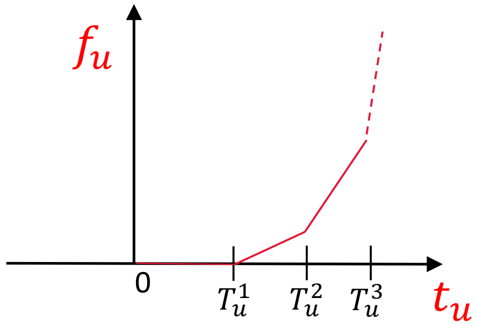
- Find a schedule t^* satisfying all fixed and disjunctive precedence constraints.
- Minimize $f(t^*)$ (deviation from T)

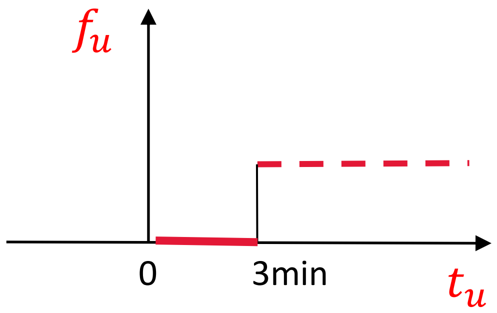
PS. Fixed route case.

}

On the objective function $\boldsymbol{f(t)}$

- Typically computed in special events, i.e. the arrival time at some stations $V^* \subset V$
- $f(t) = \sum_{u \in V^*} f_{u(t_u)}$ is often separable
- Typically $f_{u(t_u)}$ is non-decreasing.





}

Disjunctive formulation

$$\min f(t)$$

$$t_v - t_u \geq l_{uv} \qquad (u, v) \in E$$

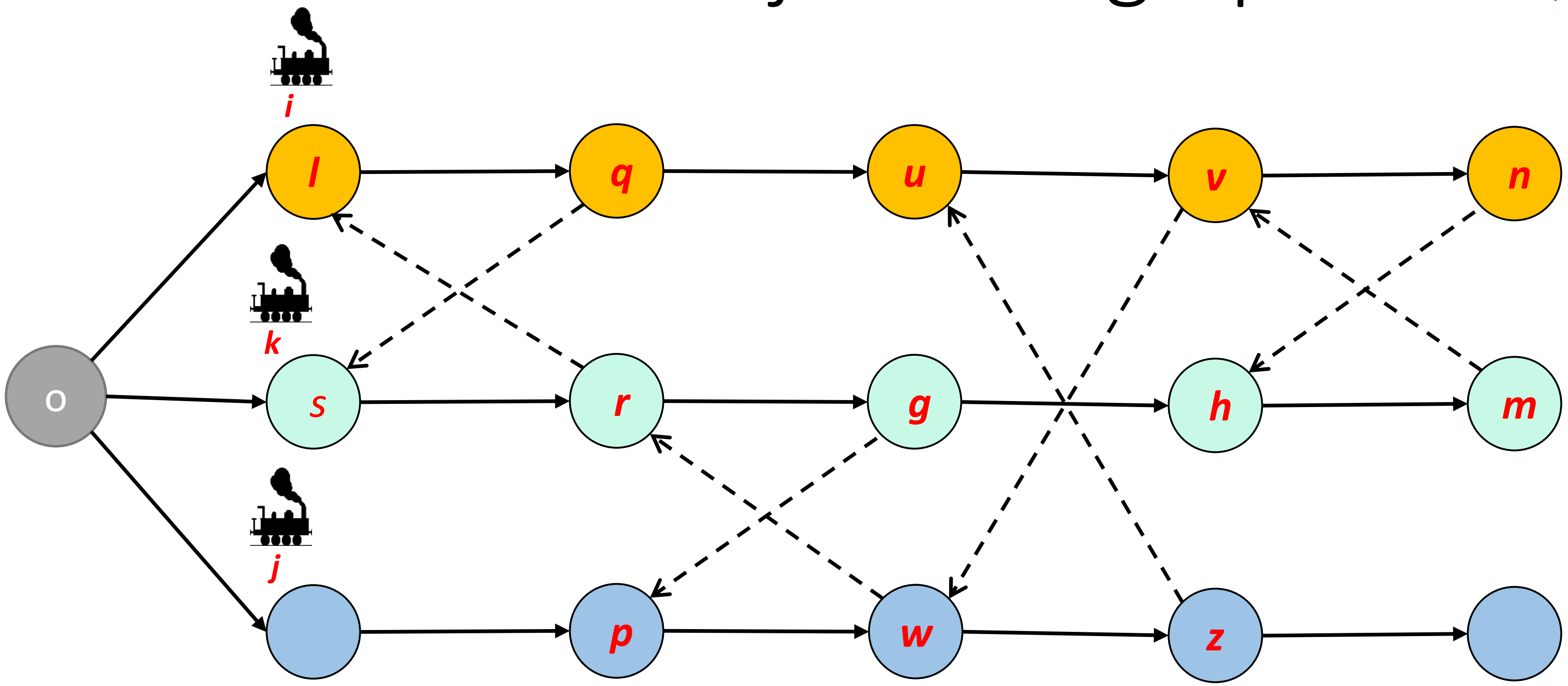
$$t_w - t_v \geq 0 \ \textbf{OR} \ t_u - t_z \geq 0 \qquad \{(v, w), (z, u)\} \in D$$

$$t \in R^V$$

- V set of events ($v \in V$ is a certain train entering a certain block or the origin), E set of precedence constraints, D set of disjunctive precedence constraints

- Train scheduling is a *job-shop* scheduling problem with blocking and no-wait constraints, *Mascis & Pacciarelli* (2002)

Disjunctive graph $G = (V, E \cup E^D)$



$\min f(t)$

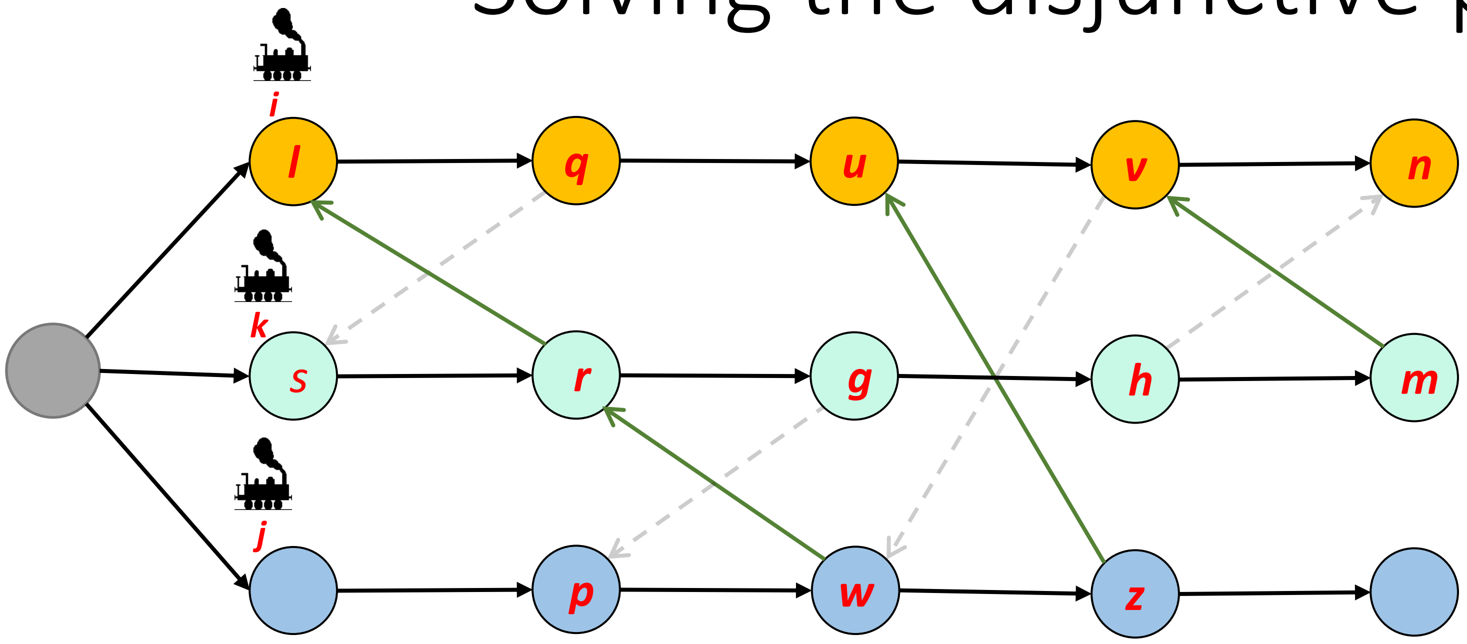
$t_v - t_u \geq l_{uv} \quad (u,v) \in E$

$t_w - t_v \geq 0$ **OR** $t_u - t_z \geq 0 \quad \{(v,w),(z,u)\} \in D$

$t \in R^V$

- ❑ V nodes (events), E directed edges, D disjunctive arcs (pairs of "*conflict*" edges E^D)
- ❑ Each conflict edge corrsponds to a specific term in a specific disjunction

Solving the disjunctive problem



$$\min f(t)$$
$$t_v - t_u \geq l_{uv} \qquad (u,v) \in E$$
$$t_w - t_v \geq 0 \textbf{ OR } t_u - t_z \geq 0 \qquad \{(v,w),(z,u)\} \in D$$
$$t \in R^V$$

$$G = (V, E \cup E^D)$$

- For each disjunction, we must decide which term is satisfied by the solution t
- Equivalent to picking exactly one (conflict) edge for each disjunctive arc
- The set of conflict edges "picked" up is called (*complete*) *selection*.

}

Big-M formulation

$$\min f(t)$$

$$\left.\begin{array}{l} y_{vw} + y_{zu} = 1 \\ t_w - t_v \geq -M(1 - y_{vw}) \\ t_u - t_z \geq -M(1 - y_{zu}) \end{array}\right\} \quad \begin{array}{ll} t_v - t_u \geq l_{uv} & (u,v) \in E \\ t_w - t_v \geq 0 \ \textbf{OR} \ t_u - t_z \geq 0 & \{(v,w),(z,u)\} \in D \\ t \in R^V, \ y \in \{0,1\}^{2D} & \end{array}$$

- ❑ Two binary (*selection*) variables y_{vw}, y_{zu} for each disjunction $\{(v,w),(z,u)\} \in D$
- ❑ And the "big-M trick"!

}

Big-M formulation

$$\min f(t)$$

Fixed precedence $\quad t_v - t_u \geq l_{uv} \quad (u,v) \in E$

Disjunctive constraints $\quad \begin{cases} t_w - t_v \geq -M(1 - y_{vw}) \\ t_u - t_z \geq -M(1 - y_{zu}) \end{cases}$

Selection constraints $\quad y_{vw} + y_{zu} = 1 \quad \{(v,w),(z,u)\} \in D$

$$t \in R^V, \; \mathrm{y} \in \{0,1\}^{2D}$$

- ❑ Big-M formulations most used in the literature on train dispatching
- ❑ An alternative: time-indexed formulations (often used in train timetabling)

Def. Feasible selections: $Y = \{y \in \{0,1\}^{2D}: y_{vw} + y_{zu} = 1, \{(v,w),(z,u)\} \in D\}$

}

Big-M formulation

$$\min f(t)$$

Fixed precedence $\quad t_v - t_u \geq l_{uv} \quad (u,v) \in E$

Disjunctive constraints
$$t_w - t_v \geq -M(1 - y_{vw})$$
$$t_u - t_z \geq -M(1 - y_{zu}) \quad \{(v,w),(z,u)\} \in D$$

$$t \in R^V, \; y \in Y$$

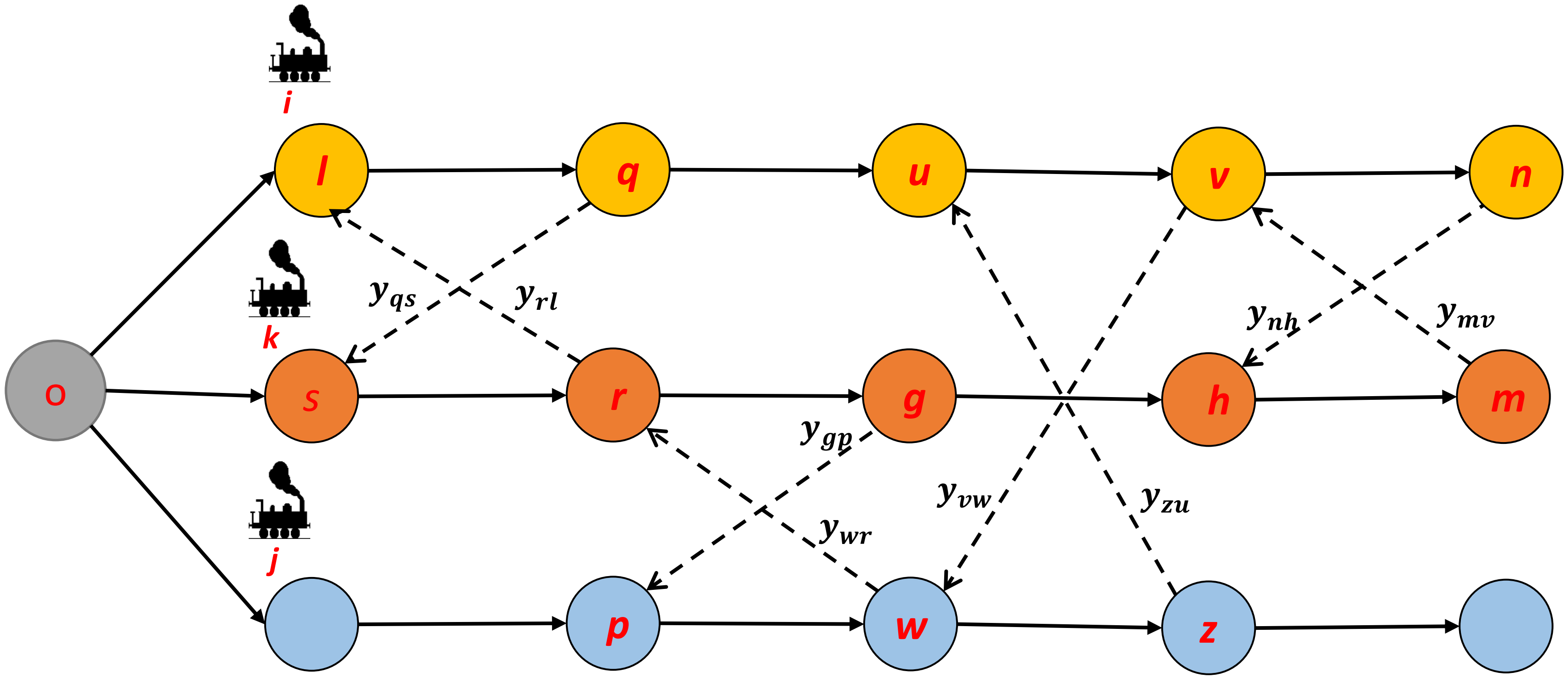
For a given selection: $\bar{y} \in Y$ let $S(\bar{y})$ be the set of selected terms. The problem becomes:

$$\min \; \{f(t): t_v - t_u \geq l_{uv}, uv \in E \cup S(\bar{y}), t \in R^V\} \qquad \text{Sched}(\bar{y})$$

- ❑ Dual of a min-cost flow problem when $f(t)$ is linear.

Benders’ decomposition(s)

Conflict edges



$\min f(t)$

$t_v - t_u \geq l_{uv} \qquad (u, v) \in E$

$t_w - t_v \geq -M(1 - y_{vw})$
$t_u - t_z \geq -M(1 - y_{zu}) \qquad \{(v, w), (z, u)\} \in D$

$t \in R^V, \quad y \in Y$

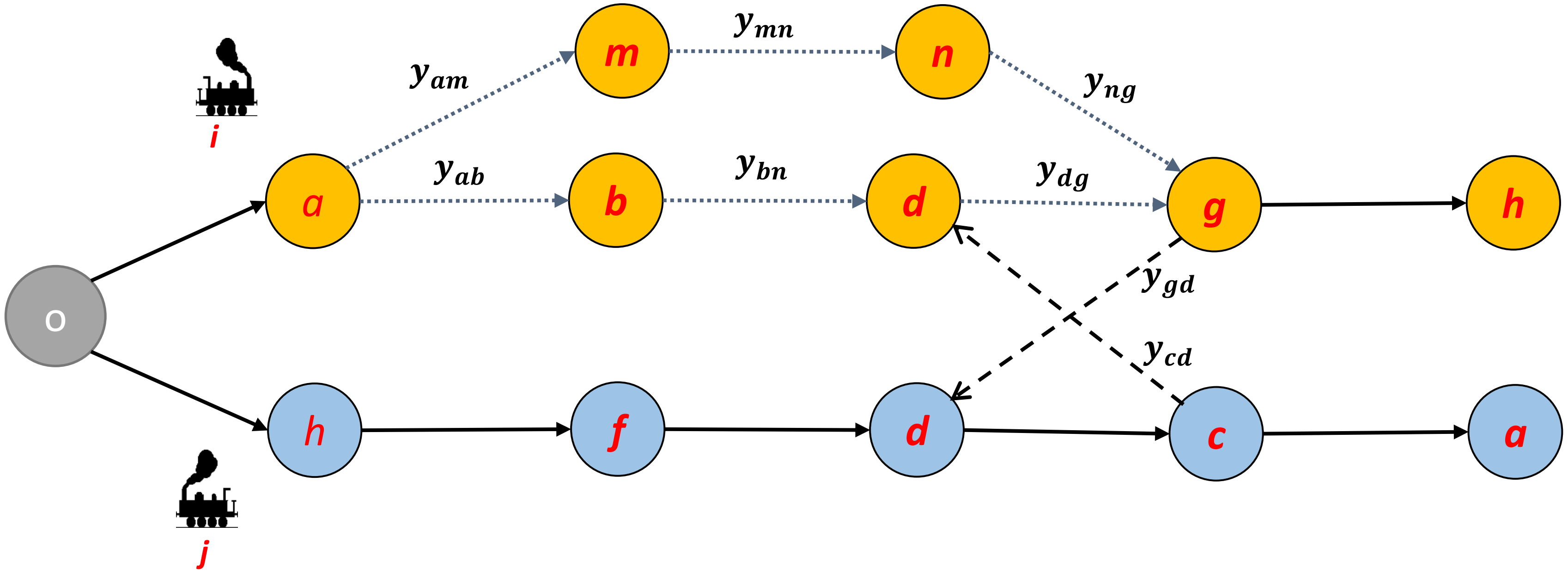
$G = (V, E \cup E^D)$

- Each conflict edge $e \in E^D$ is associated with a selection variable y_e
- $y \in Y$ is the incidence vector of a set $S(y) \subseteq E^D$ of (*conflict*) edges

26

What to do with routing?

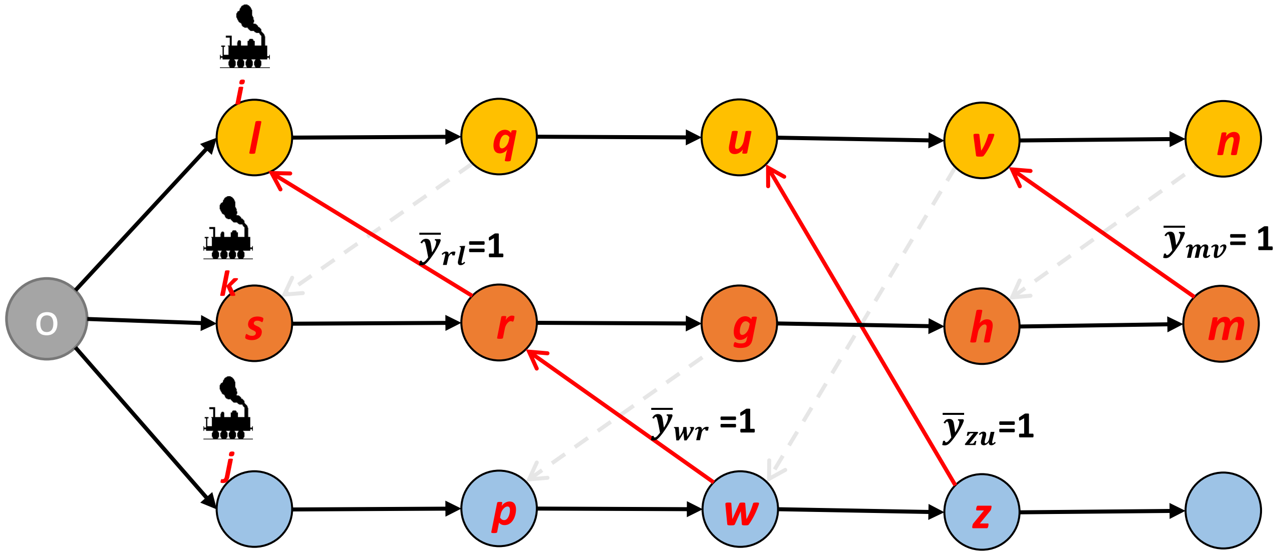
- Add the alternative *routing edges* E^R and binary (*routing*) variables $y_e, e \in E^R$



- Extend the set Y: new variables, multicommodity flow and coupling costraints.

Disjunctive graph and scheduling

- For $\bar{y} \in Y$ the disjunctive graph becomes a standard graph $G(\bar{y}) = (V, E \cup S(\bar{y}))$



- How does $G(\bar{y})$ relate to the associated scheduling problem Sched($\bar{y}$)?

Sched($\bar{y}$)

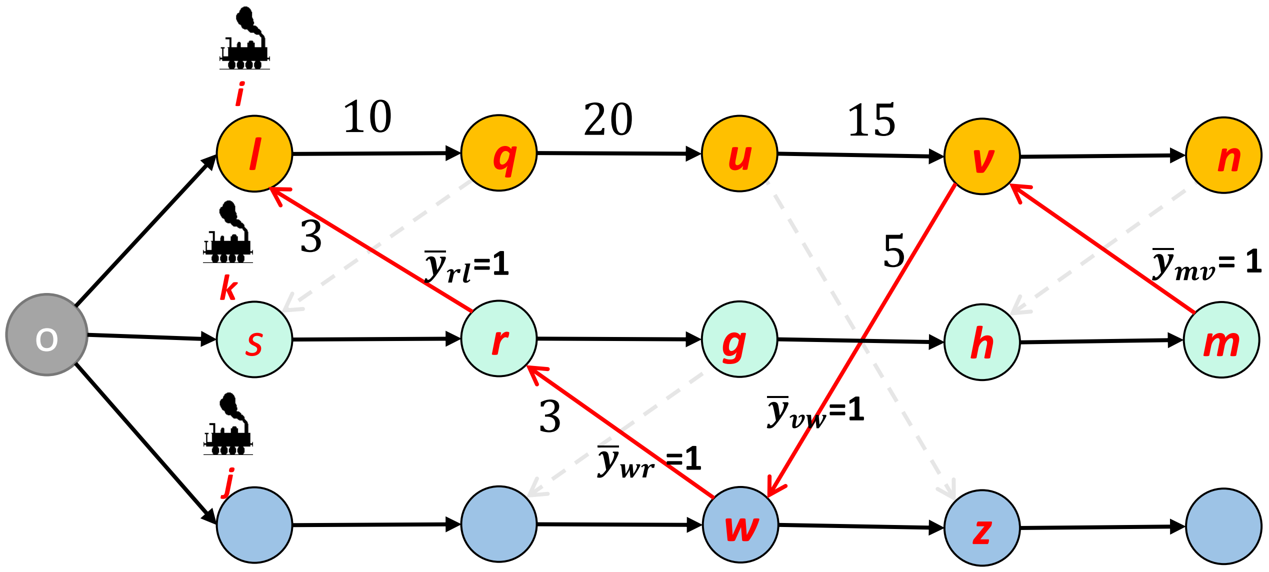
$$\min f(t)$$

$$t_v - t_u \geq l_{uv}, uv \in E \cup S(\bar{y})$$

$$t \in R^V$$

Feasibility

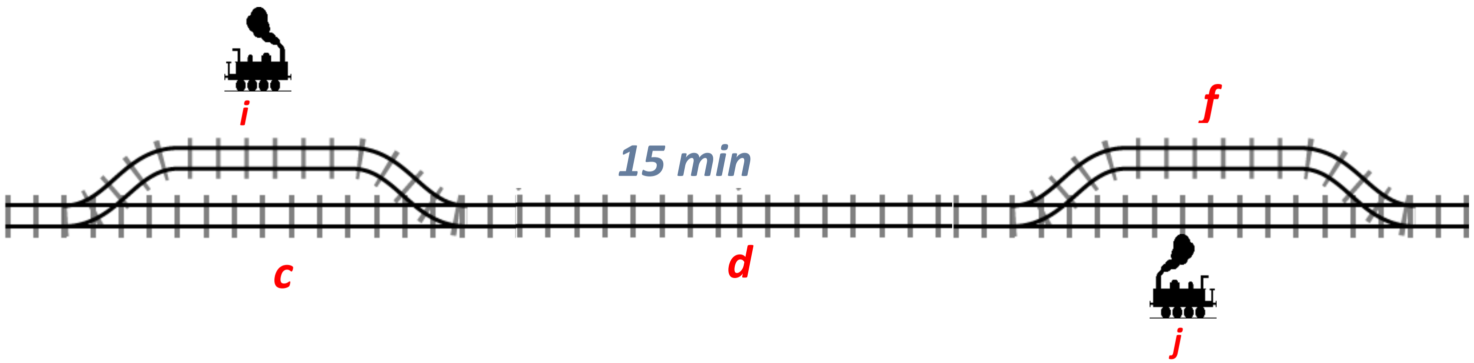
Th. 1. For $\bar{y} \in Y$, Sched($\bar{y}$) has a solution, if and only if $G(\bar{y})$ does not contain a directed cycle C of positive length $l(C)$.

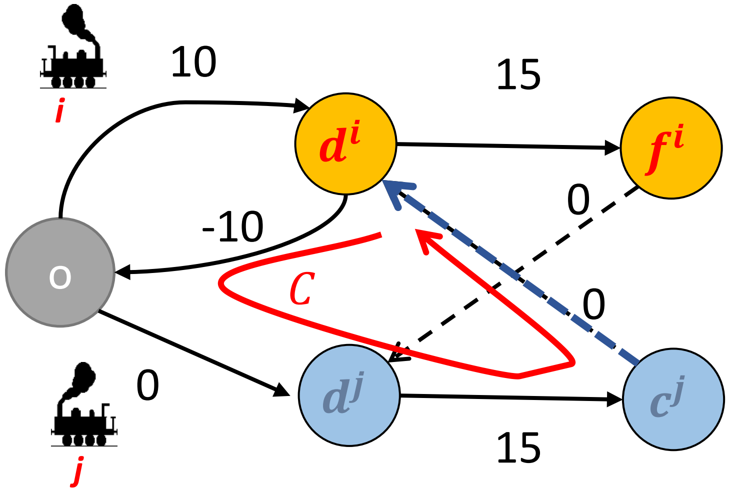


$C = \{(lq), (qu), (uv), (vw), (wr), (rl)\}$

$l(C) = 10 + 20 + 15 + 5 + 3 + 3 > 0$

Example of infeasible solution





Current time is **09:00**

Train *i* leaves the station **at 9:10** (exactly)

Train *j* can leave the station at any time from now

Suppose j wins the conflict on d

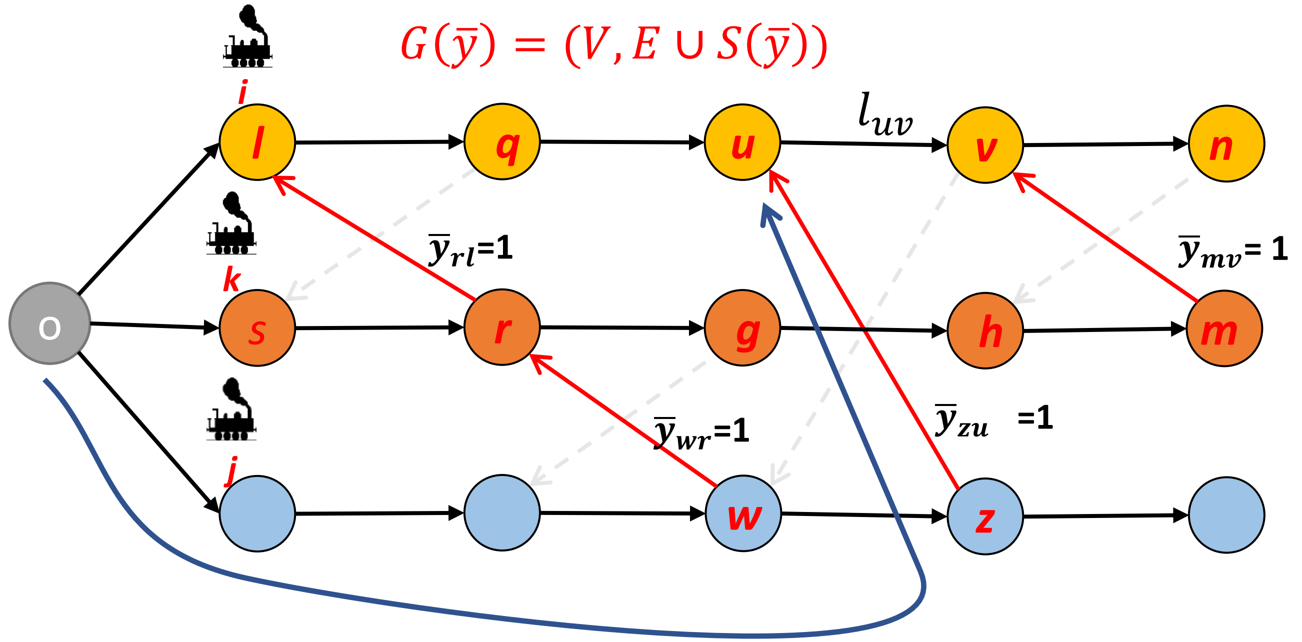
j wins → pick edge (c^j, d^i)

Cycle $C = \{c^j d^i, d^i\, o, o d^j, d^j c^j\}$, **length 5 > 0**

Feasible solutions

- ❑ $\bar{y} \in Y, G(\bar{y})$ no positive directed cycles.

Th. 2. $\bar{t}_u^* =$ length of longest path from o to $u \in V$ in $G(\bar{y})$ is feasible for Sched($\bar{y}$)



Sched($\bar{y}$)

$$\min f(t)$$

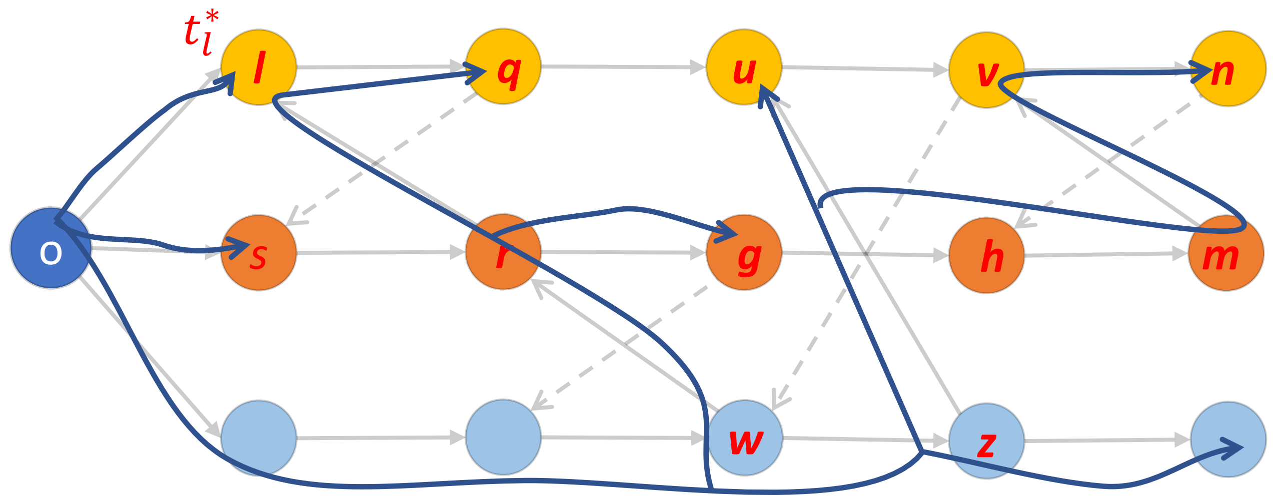
$$t_v - t_u \geq l_{uv}, uv \in E \cup S(\bar{y})$$

$$t \in R^V$$

Optimal solutions

- ❑ $\bar{y} \in Y, G(\bar{y})$ no positive dicycles. For $u \in V$, $\bar{t}^*_u =$ length of longest ou-path in $G(\bar{y})$

Th. 3. *If* $f(t)$ is non-decreasing then $\bar{t}^*$ is an optimal solution for Sched($\bar{y}$)



Sched($\bar{y}$)

$$\min f(t)$$

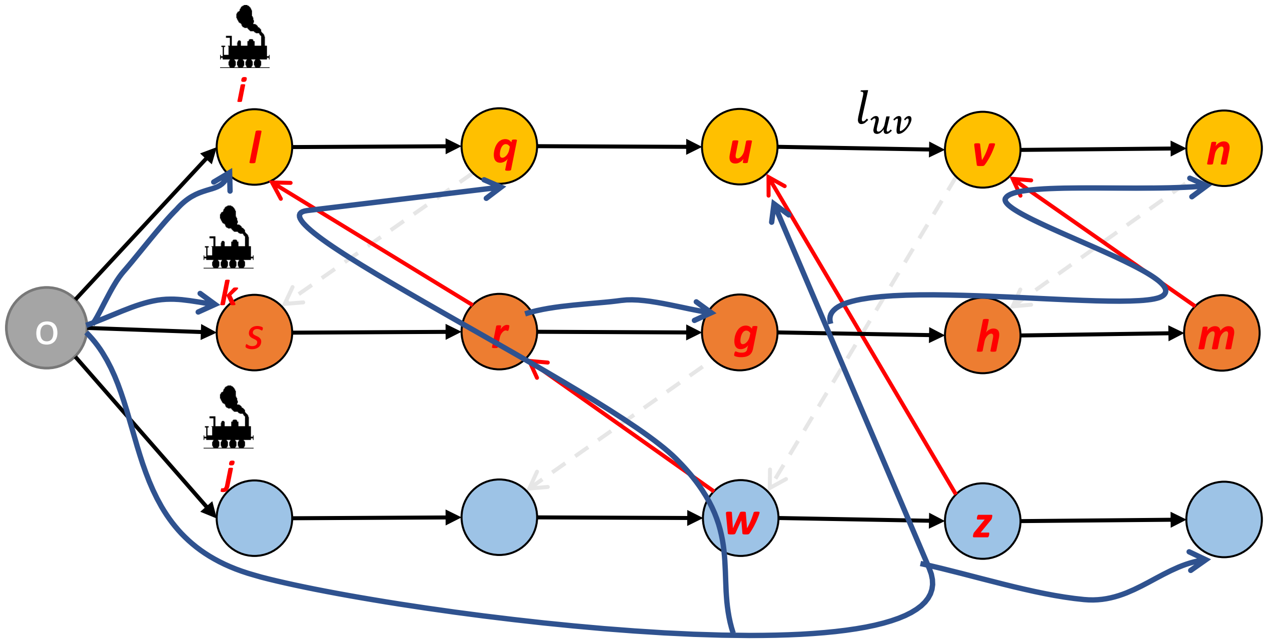
$$t_v - t_u \geq l_{uv}, uv \in E \cup S(\bar{y})$$

$$t \in R^V$$

Def. $H^*(\bar{y})$ longest path tree in $G(\bar{y})$ then let $c(H^*) = f(\bar{t}^*)$ be the cost of H^*

Scheduling problem for regular $\boldsymbol{f(t)}$

Find $\bar{y} \in Y$, such that $G(\bar{y})$ has no positive directed cycles and the cost $c(H^*)$ of a longest path tree H^* in $G(\bar{y})$ is minimized.

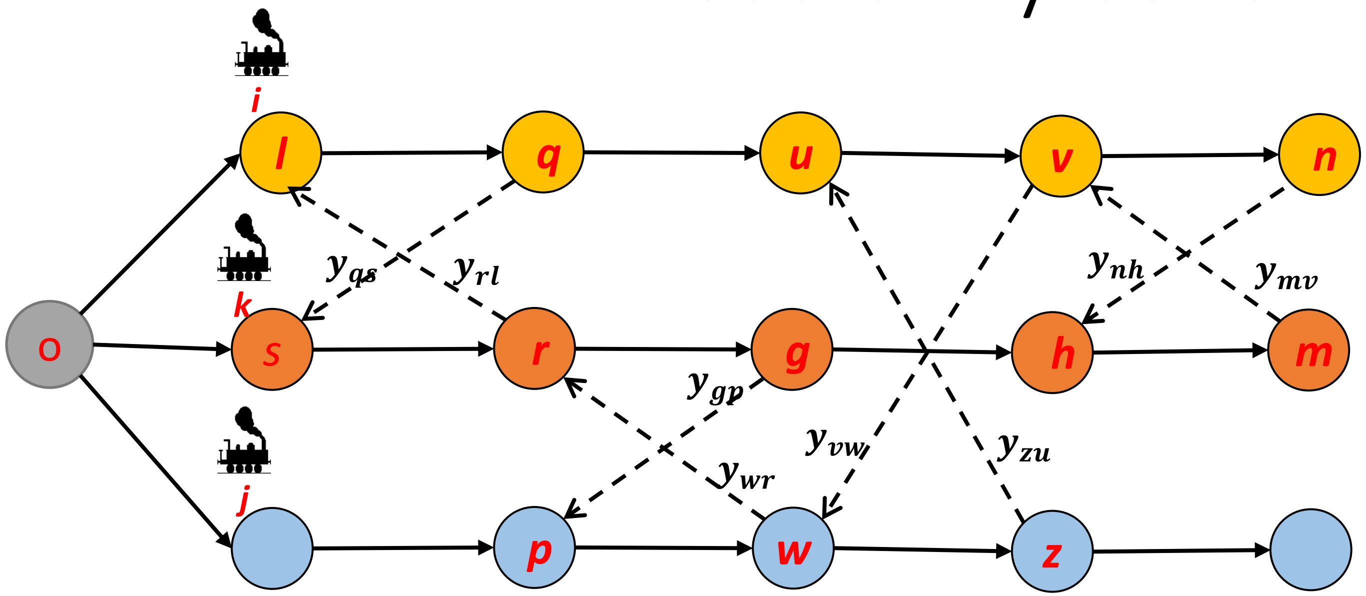


$G(\bar{y})$

The Path&Cycle formulation

- This led to a new (*Path&Cycle*, 2019) formulation without annoying big-M constraints (but potentially many constraints)
- Based on disjunctive graph $G = (V, E \cup E^D)$ $(G = (V, E \cup E^D \cup E^R))$
- Binary variables y_e for $e \in E^D$ $(e \in E^D \cup E^R)$,
- One real variable μ representig the objective value.
- Two types of constraints: *feasibility* and *optimality*
- Feasibility constraints correspond to the positive lengths directed cycles of G
- Optimality constraints correspond to longest path trees of G.

Feasibility constraints

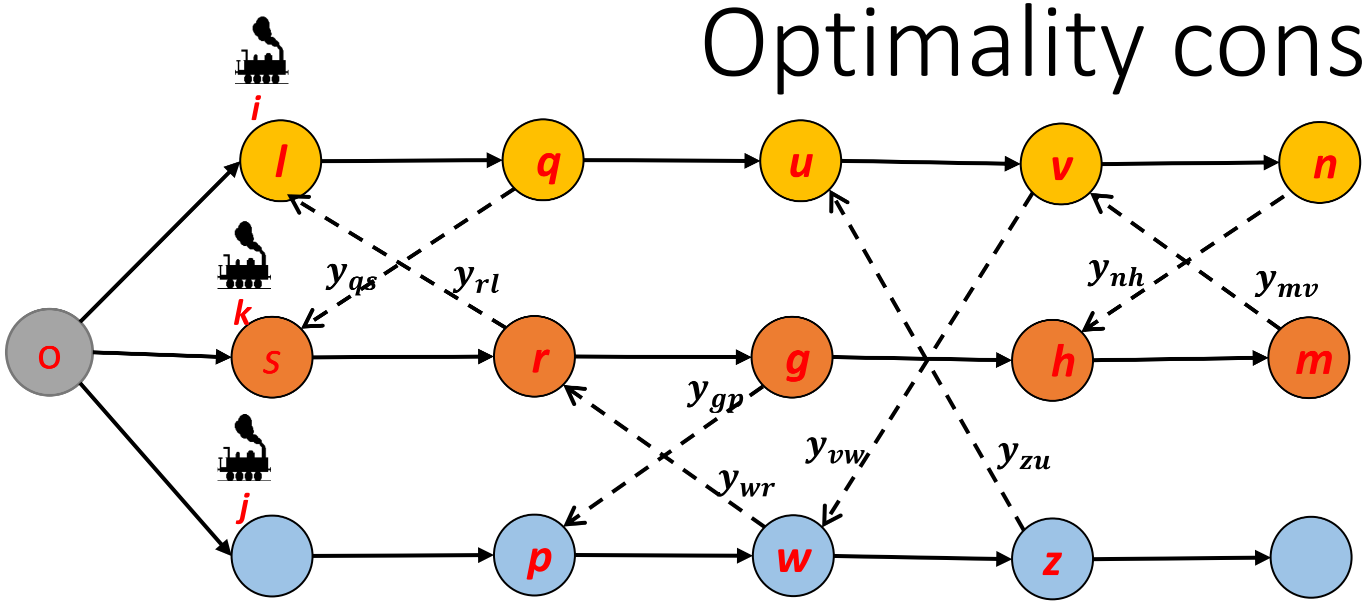


$G = (V, E \cup E^D)$

Let Ω^+ be the set of positive directed cycles of $G = (V, E \cup E^D)$

Feasibility constraint $\sum_{e \in C^D} y_e \leq |C^D| - 1,$ for $C^D = E^D \cap C, C \in \Omega^+$

Optimality constraints



$G = (V, E \cup E^D)$

η cost of solution

$$Y^+ = \left\{ y \in Y: \sum_{e \in C \cap E^D} y_e \le |C^D| - 1, \text{for } C^D = C \cap E^D, C \in \Omega^+ \right\}$$

$$\Pi^* = \{H^*(y) \textit{ longest path tree in } G(y): y \in Y^+\}$$

Optimality cuts

$$\eta \ge c(H)\left(\sum_{e \in H^D} y_e - |H^D| - 1\right), \quad \text{for } H^D = H \cap E^D, H \in \Pi^*$$

The Path and Cycle formulation

min η

Feasibility $\sum_{e \in C^D} y_e \leq |C^D| - 1,$ for $C^D = E^D \cap C, C \in \Omega^+,$

Optimality $\eta \geq c(H)\left(\sum_{e \in H^D} y_e - |H^D| - 1\right),$ for $H^D = H \cap E^D, H \in \Pi^*$

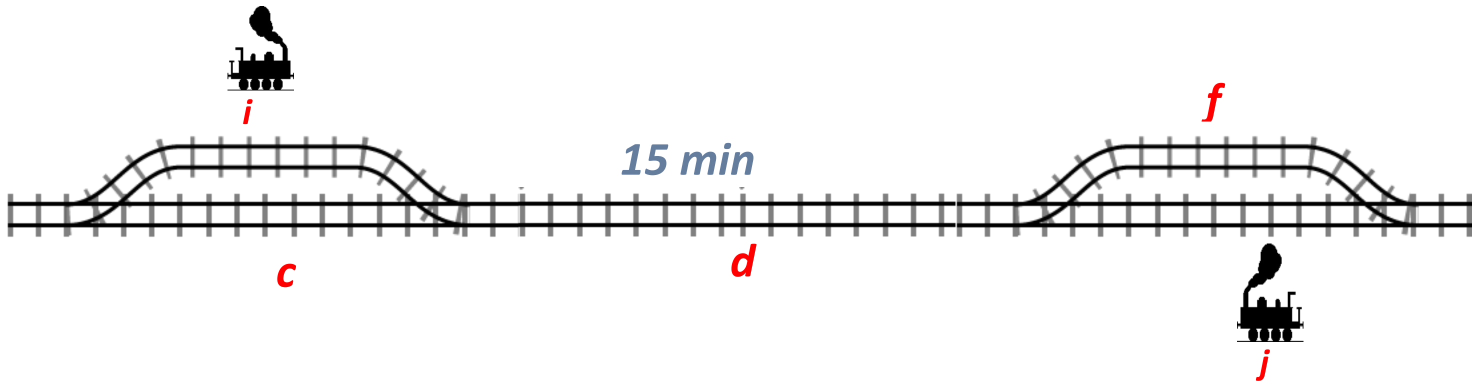
$\eta \in R, \quad y \in \{0,1\}^{E^D}$

- ❑ Many constraints: solve by delayed row generation

Problem infeasible → there exists a family $\bar{\Omega} \subseteq \Omega^+$ of positive directed cycles $G = (V, E \cup E^D)$ such every $y \in Y$ «contains» at least a cycle in $\bar{\Omega}$:

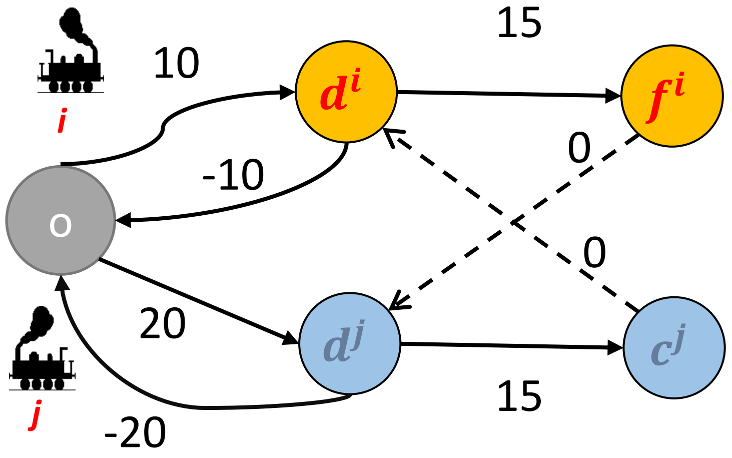
For $y \in Y, \exists C^y \in \bar{\Omega}$ such that $S(y) \cap C^y = E^D \cap C^y$

An example



Current time is **09:00**
Train *i* must leave **at 9:10**
Train *j* must leave **at 9:20**

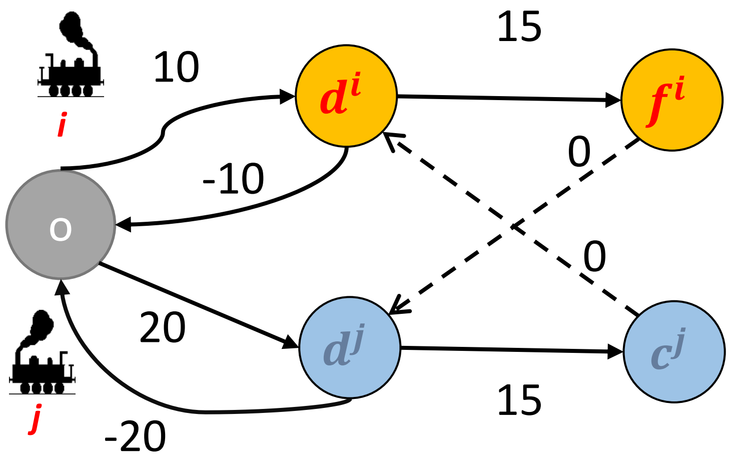
$G = (V, E \cup E^D)$



Disjunctive graph representation

Infeasibility proof

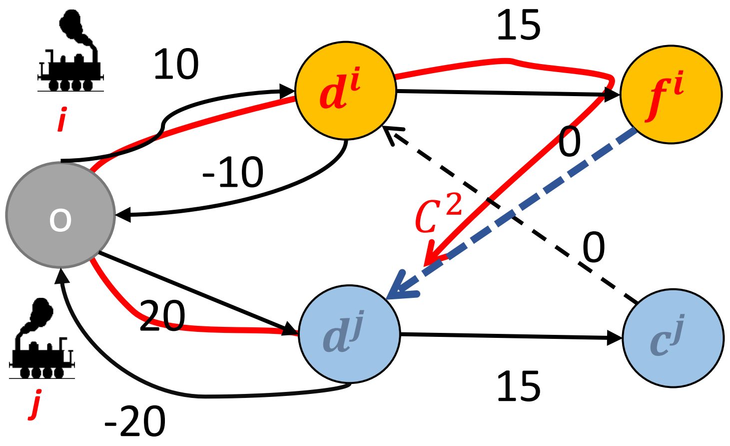
$G = (V, E \cup E^D)$



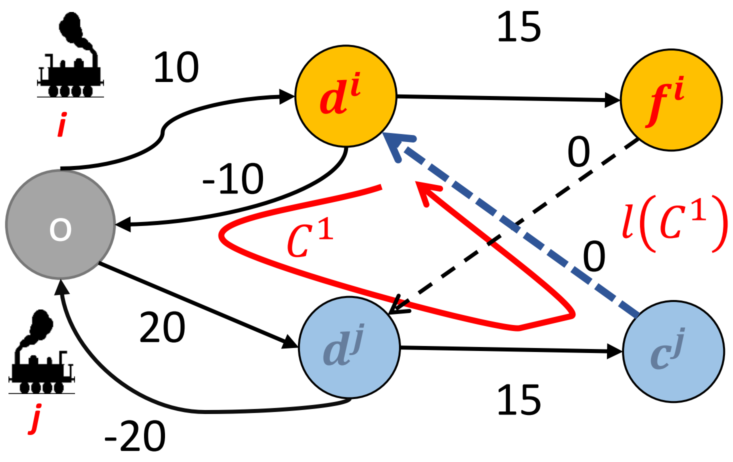
- Problem infeasible: every $y \in Y$ «contains» a cycle in $\overline{\Omega} = \{C^1, C^2\}$:

$$S(y) \cap C^y = E^D \cap C^y, for\ some\ \ C^y \in \overline{\Omega}$$

$l(C^2) = +5$



i wins



$l(C^1) = +25$

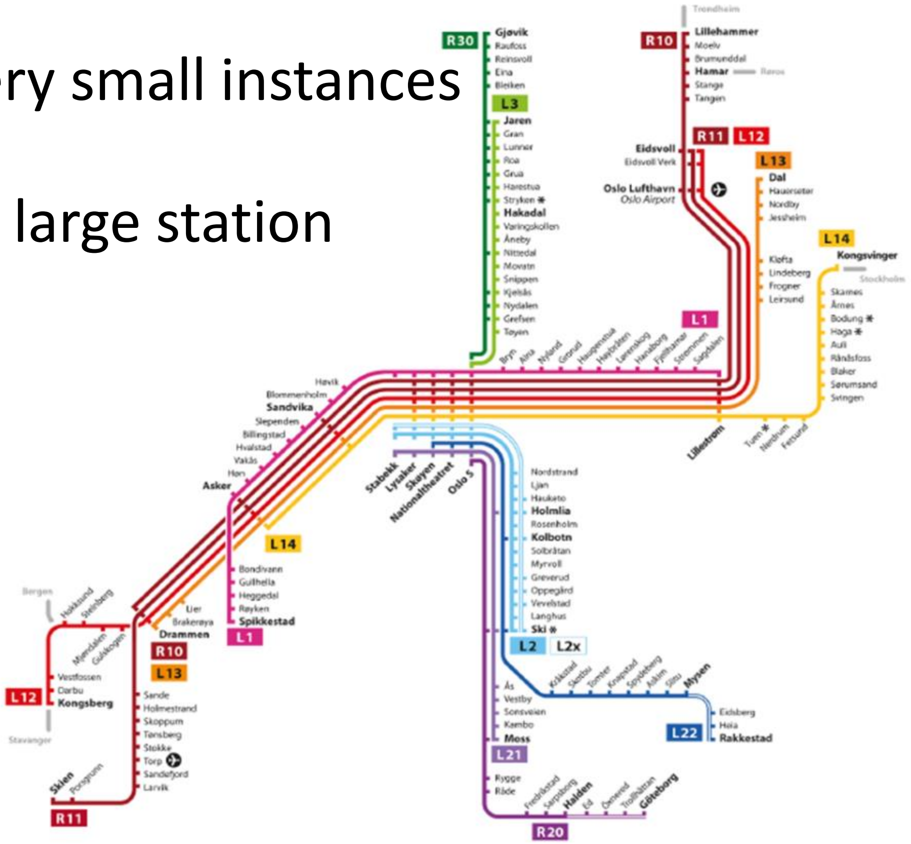
j wins

$$\overline{\Omega} = \{C^1, C^2\}$$

A real-life pilot application

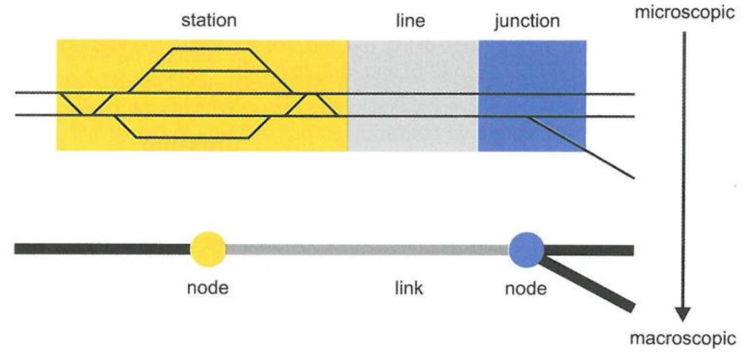
Greater Oslo Area Railway

- We can solve the *Big-M* or P&C formulations for very small instances
- Greater Oslo Area Railway is a combination of one large station (Oslo S) and 10 municipal lines incident to Oslo S
- Almost 1000 trains daily
- **Need:** more decomposition/reformulation



Further decomposition

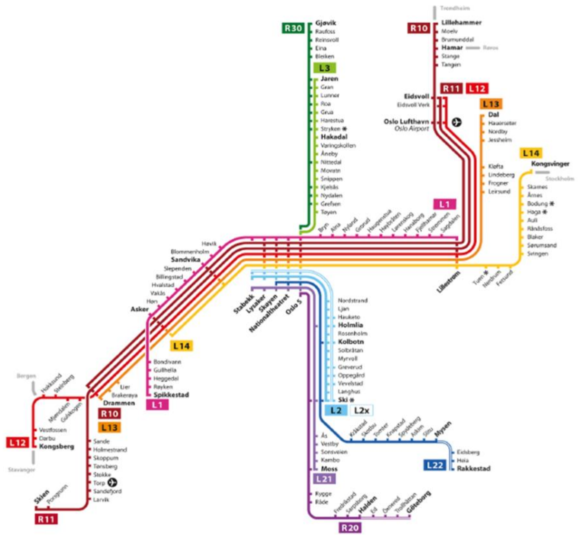
- One popular decomposition approach is the so called *Macroscopic/Microscopic* decomposition.

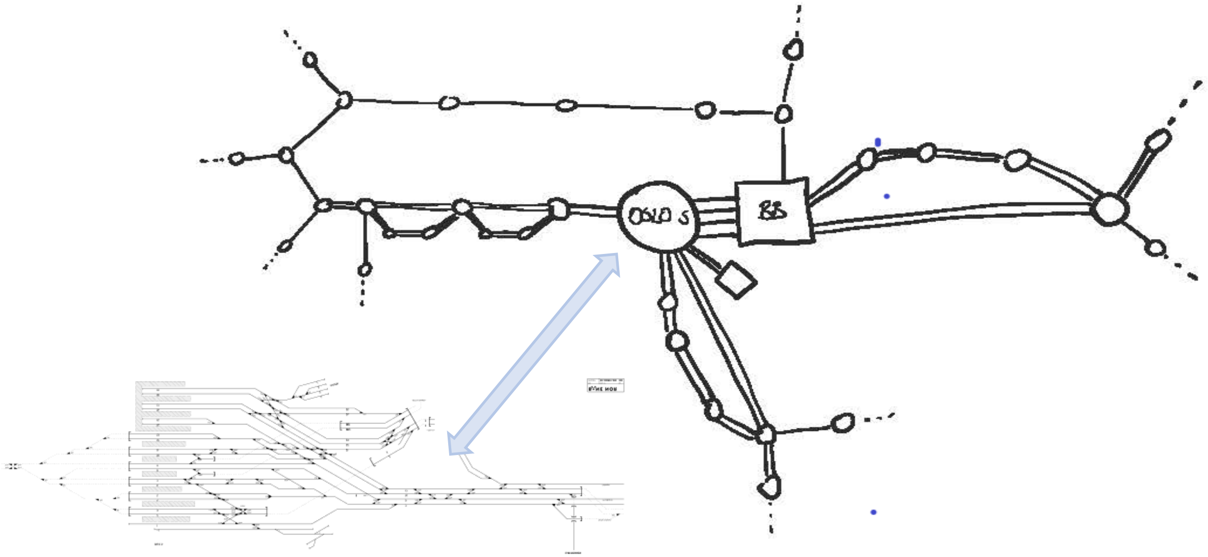


(Figure from Hansen and Pachl, *Railway Timetable & Operations*)

- Subnetworks (as stations) are collapsed into "capacited" nodes.
- A solution is found for the collapsed (macroscopic) representation
- The solution is then extended to the original re-expanded (microscopic) areas

Collapsing Greater Oslo Railway

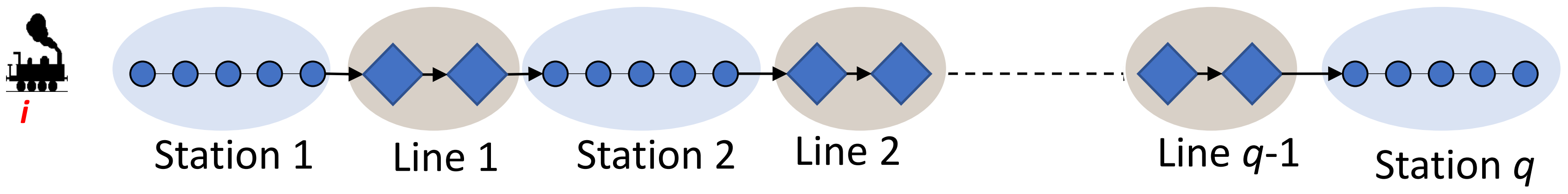




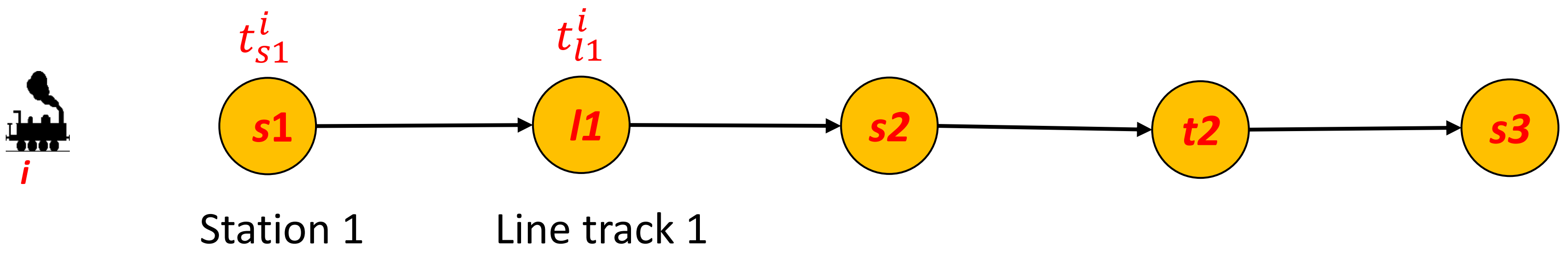
- ❑ Macroscopic solution = arrival and departure time for each train in each station (timetable)
- ❑ Can we extend the macro solution? = For each station, is the timetable feasible?

Macroscopic representation of train routes

Microscopic representation



Macroscopic representation



Block structure of constraint matrix

$$\min f(t^T)$$

$$At^L + \boxed{Bt^T} + 0 \leq b - M^L y^L$$ schedule on the line = macro network

$$0 + \boxed{Ct^T} + Dt^S \leq d - M^S y^S$$ schedule in stations = micro network(s)

y^L, y^S binary, t^L, t^T, t^S real

- Constraint matrix with quasi-block structure
- Station tracks and line tracks share only timetable variables t^T
- The objective function is only in t^T
- Station constraints decompose $\begin{pmatrix} C^1, D^1 & & & \\ & C^2, D^2 & & \\ & & C^3, D^3 & \\ & & & \dots \end{pmatrix}, \begin{pmatrix} M^1 & & & \\ & M^2 & & \\ & & M^3 & \\ & & & \dots \end{pmatrix}$

Logic Benders' Reformulation

$\min f(t^T)$

$At^L +$	Bt^T		$\leq b - M^L y^L$	schedule on the line	MASTER
$0 \quad +$	$Ct^T +$	Dt^S	$\leq d - M^S y^S$	schedule in stations	SLAVE

y^L, y^S binary, t^L, t^T, t^S real

Reformulation

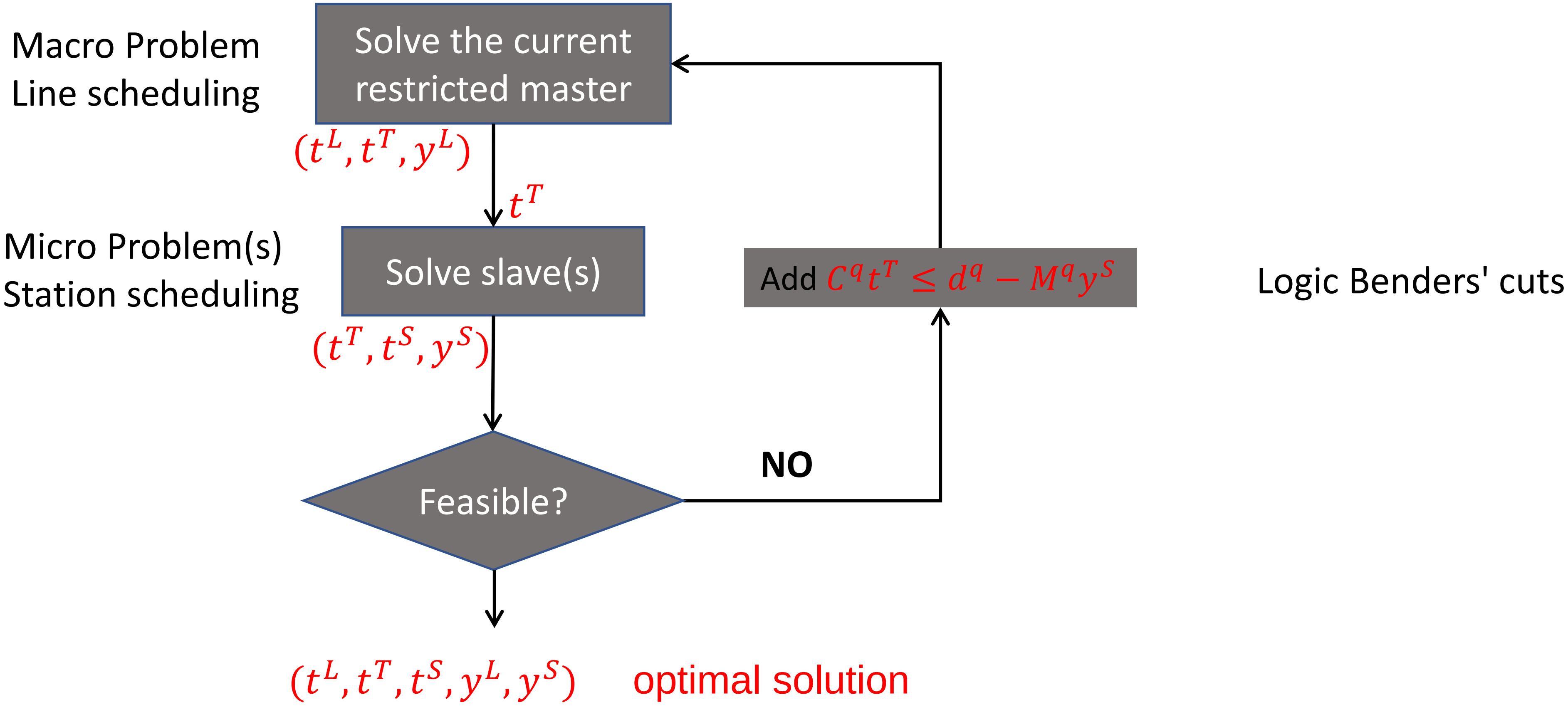
$\min f(t^T)$

$At^L +$	Bt^T	$\leq b - M^L y^L$	schedule on the line	
	$C't^T$	$\leq d' - M'^S y^S$	logic Benders' cuts	

$y = (y^L, y^S)$ binary, t^L, t^T real

Solving the Train Scheduling Problem

- ❑ Apply row generation



The slave feasibility problem

- The slave problem decomposes in many independent feasibility problem

Station feasibility problem:

A. Given a station and arrival and departure times for all trains (a timetable), does a feasible solution (in the station) exist?

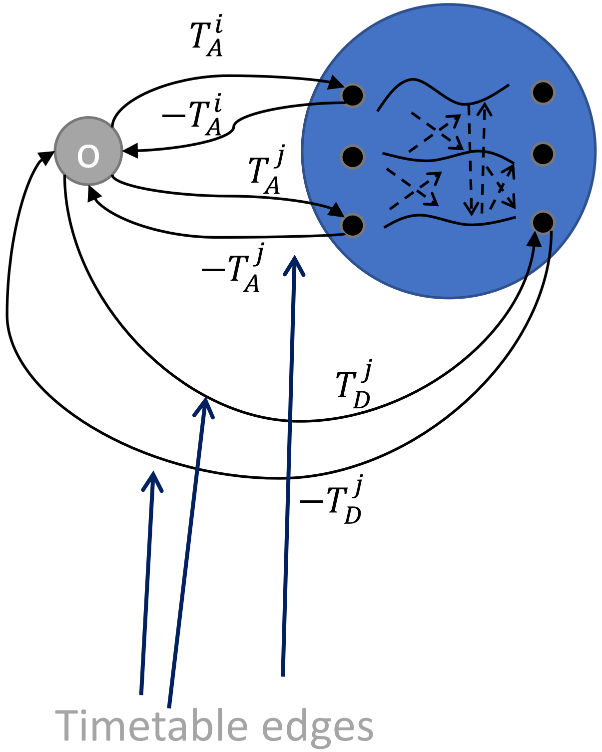
B. If the problem is infeasible, what are the constraints to return to the master?

- We exploit the feasibility conditions of the P&C formulation

Individual station problem

- Station problem: given arrival times $T_A^1,\ T_A^2, \dots,$ departure times $T_D^1,\ T_D^2, \dots,$ does there exist a feasible solution?

$G = \left(V, E \cup E^D \cup E^R\right)$ disjunctive graph representing problem instance



$Y = \{y^1, y^2, \dots\}$ set of (incident vectors of) edge selections

Station problem infeasible:

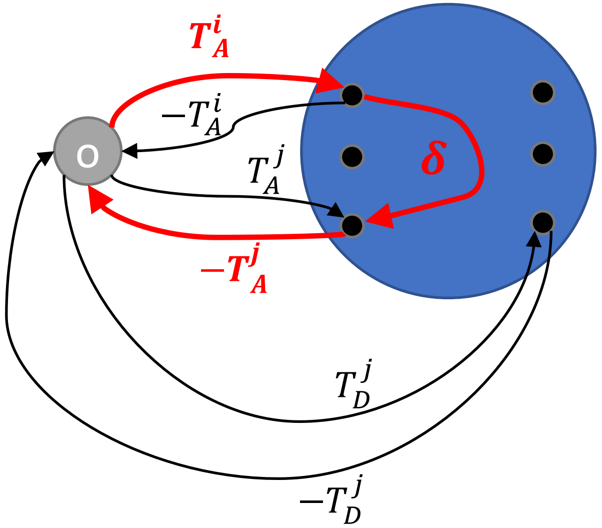
G contains a family $\overline{\Omega} = \{C^1, C^2, \dots\}$ of positive lengths cycles such that every selection $y \in Y$ "contains" a cycle, i.e.

$$S(y) \cap C^i = E^D \cap C^i, \qquad for\ some\ \ C^i \in \overline{\Omega}$$

Combinatorial Benders' cuts

$\bar{\Omega} = \{C_1, C_2, \dots\}$. Suppose $C \in \bar{\Omega}$ contains a timetable edge.

Then C contains the origin o and exactly two timetable edges.



C timetable cycle

$$l(C) > 0 \rightarrow T_A^i - T_A^j + \delta > 0 \rightarrow T_A^j - T_A^i < \delta$$

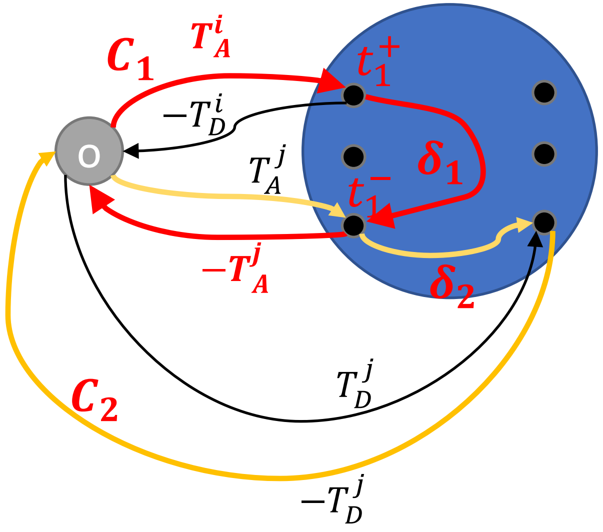
To prevent $l(C) > 0$ *a* timetable must satisfy

$$t_A^j - t_A^i \geq \delta$$

Combinatorial Benders' cuts

$\overline{\Omega}$: every selection $y \in Y$ *"contains"* a cycle in $\overline{\Omega}$

$\Omega^T \subseteq \overline{\Omega}$ subset of "timetable" cycles of $\overline{\Omega}$



For $C_i \in \Omega^T$, t_i^-, t_i^+ time variable associated with (the other endpoint of) the non-positive edge and non-negative edge,

Then, for any feasible timetable t, we must have:

$t_1^- - t_1^+ \geq \delta_1$ **OR** $t_2^- - t_2^+ \geq \delta_2$ **OR …**

Again, a disjunction of time precedence constraints!

}

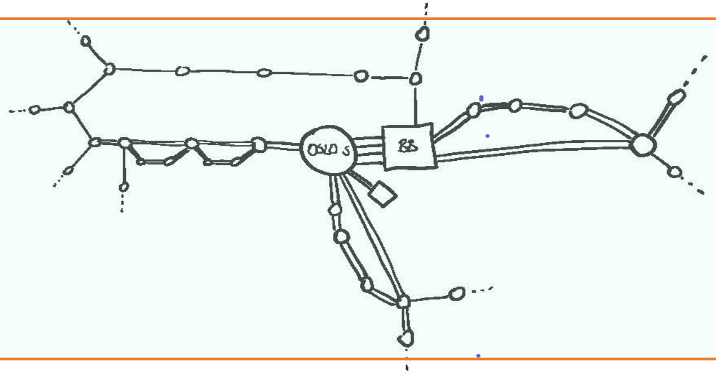
The full reformulation

$$\min f(t)$$

$$t_v - t_u \geq l_{uv} \qquad (u,v) \in E$$

other line constraints …

macro problem



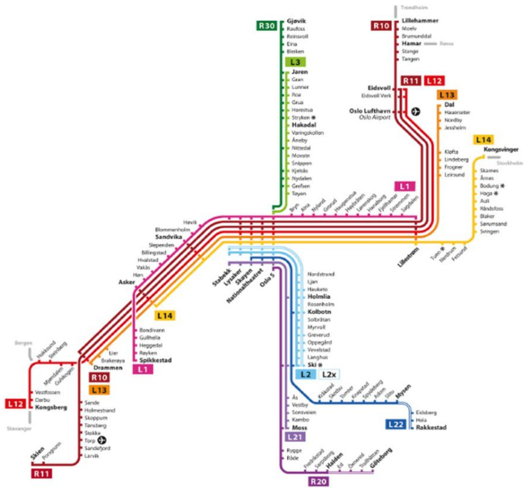
$$\bigvee_{C_i \in \Omega^k} \; t_i^- - t_i^+ \geq \delta_i \qquad \Omega^k \in \Lambda$$

logic Benders' cuts

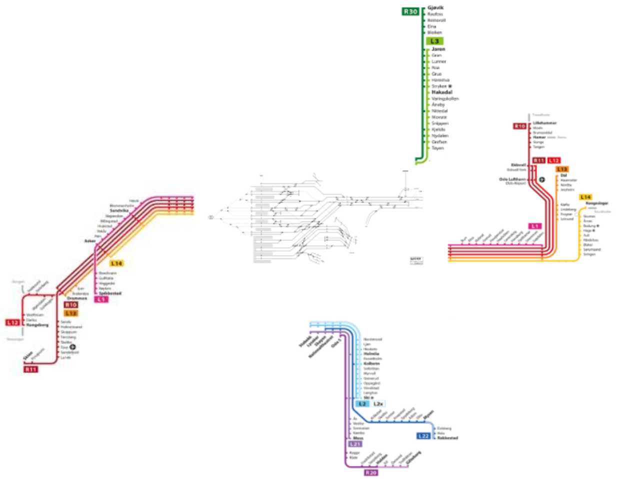
$$t \in R^V$$

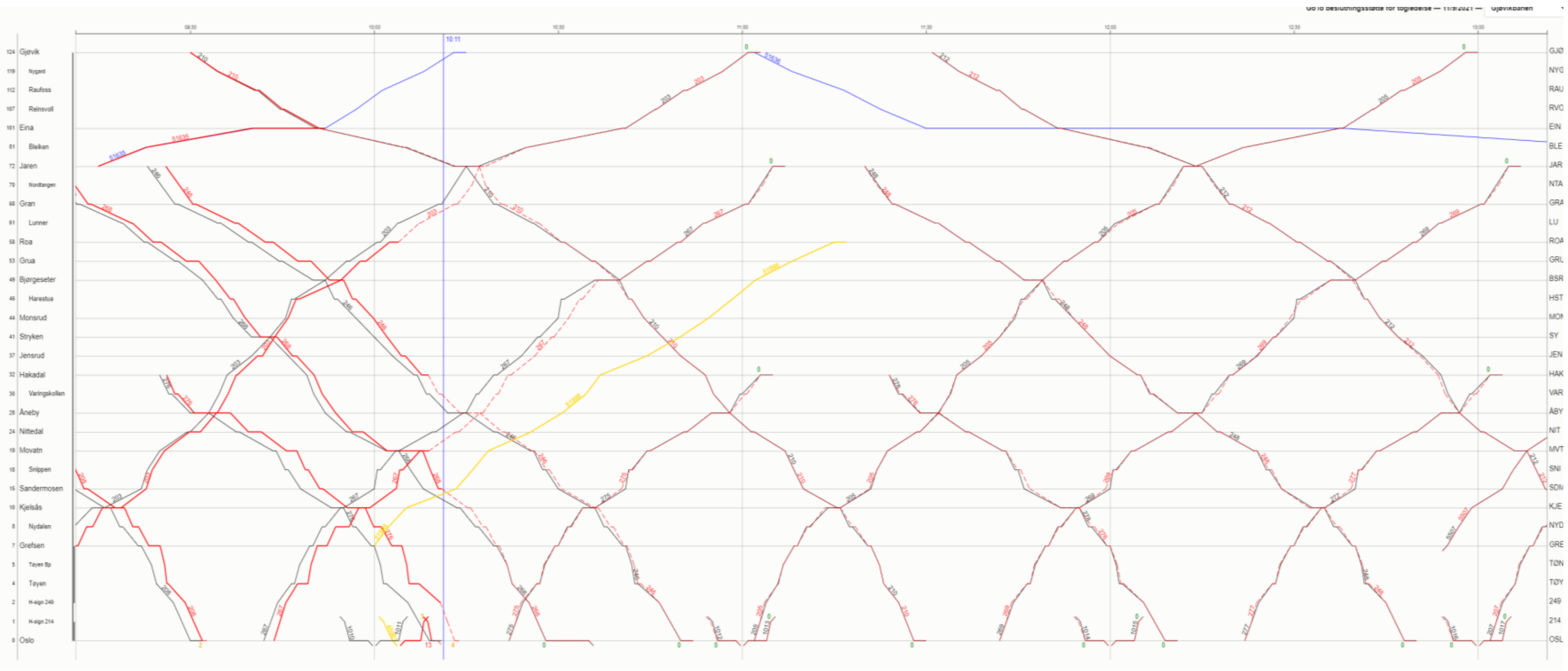
- ❑ Λ is the set of all families of timetable cycles (for all stations!)
- ❑ Disjunctions can be linearized by introducing binary variables and big Ms.

Dispatching system in Oslo



- ❑ We developed a real-time scheduling system for dispatching trains in Oslo Greater Oslo Region





References

- Balas, E. (1969). Machine sequencing via disjunctive graphs, *Operations Research* 17 (1969) pp. 941–957.

- Codato, G. and Fischetti, M., 2006. Combinatorial Benders' cuts for mixed-integer linear programming. *Operations Research*, *54*(4), pp.756-766.

- Corman, F. and Meng, L., 2014. A review of online dynamic models and algorithms for railway traffic management. *IEEE Transactions on Intelligent Transportation Systems*, *16*(3), pp.1274-1284.

- Lamorgese, L., & Mannino, C. (2015). An exact decomposition approach for the real-time train dispatching problem. *Operations Research*, *63*(1), 48-64.

- Lamorgese, L., & Mannino, C. (2019). A noncompact formulation for job-shop scheduling problems in traffic management. *Operations Research*, *67*(6), 1586-1609.

- Leutwiler, F., Corman, F. (2021). A logic Benders' decomposition for microscopic railway timetable plalnning, *EURO 2021 conference,* Athens, July 2021.

- Mascis, A., & Pacciarelli, D. (2002). Job-shop scheduling with blocking and no-wait constraints. *European Journal of Operational Research*, *143*(3), 498-517.

- Queyranne, M. and Schulz, A.S. (1994). *Polyhedral approaches to machine scheduling*. Berlin: TU, Fachbereich 3.